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Do Students Trust in Mathematics or Intuition during Physics Problem Solving? An Epistemic Game Perspective

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This study aims to investigate (1) students' trust in mathematics calculation versus intuition in a physics problem solving and (2) whether this trust is related to achievement in physics in the context of epistemic game theoretical framework. To achieve this research objective, paper-pencil and interview sessions were conducted. A paper-pencil test was administered to 83 freshmen students. In paper-pencil test students were asked to calculate accelerations of four vehicles with different masses doing a drag race and to indicate which vehicle would win the race. Analyses revealed two themes: (1) "Math does not lie!" (for 43% of students) and (2) "Intuition does not mislead!" (for 57% of students). Interview analysis revealed that students' trust in mathematics calculation stems from the convincing and realistic nature of calculations and the misleading nature of intuitive knowledge. It was found that students' trust in intuitive knowledge stems from their intuitive expectations. The findings also revealed a significant association between students' trust in mathematics calculation and their achievement in the general physics course.

Keywords: Epistemic Games, Intuitive Knowledge, Mathematical Calculations, Physics Education, Students' Achievement

INTRODUCTION

Epistemology is defined as "an area of philosophy concerned with the nature and justification of human knowledge" (Hofer & Pintrich, 1997, p. 88). Epistemology is concerned with students' beliefs about learning and knowing and the influence of these beliefs on the cognitive processes such as thinking and reasoning (Hofer & Pintrich, 1997). Hammer (1994, p. 151) indicated that "students' epistemological beliefs may have a significant effect on how they approach the material and on what they learn". Epistemological beliefs are also considered to be influence problem-solving performance (Jonassen, 2000). This means that procedural, declarative knowledge, and problem solving strategies do not account for good problem solving; the epistemological beliefs of problem solvers must be considered.

Hammer (1994) developed a framework to characterise students' epistemological beliefs. One of the dimensions of this framework is content of knowledge which includes formula-centred and conceptual knowledge. This dimension puts physics formulas at one end and concepts at the other end. Formulas are problem solving activities and they mean that "physics knowledge is thought to consist of facts, formulas, and procedures" (Hammer, 1994, p. 158). As for conceptual knowledge, concepts refer to "informal knowledge" to include intuition (evolved from

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State of the literature

- Epistemological beliefs of students have significant effects on their learning and determine their performance during problem solving.
- Intuitive knowledge can be included in traditional physics problems in textbooks. Elby pair questions involve the exercise of a preference when solving a physics problem which makes it possible to determine the consistency or inconsistency in the results of students' problem solving.
- Researches indicate that epistemological framework is useful for investigating students' understanding and use of mathematics in physics. While the term 'epistemic' means construction of new knowledge to make sense of phenomena in the world and "game" means involvement of strategies and moves.

Contribution of this paper to the literature

- The existing surveys such as the MPEX and CLASS only assess students' epistemological beliefs about physics but do not examine qualitatively the impact of trust in calculation during problem solving and the reasons behind the trust students have. This study aims to investigate how trust in mathematical calculation and intuition influences coherence in problem solving.
- The study examine how to use epistemic game theoretical framework for analyzing students' preferences concerning problem solving strategies.
- The study also investigates students' decisions about which epistemic games to be played during a physics problem.

experience and conceptual knowledge based on a qualitative sense of principles or structure)" (Hammer, 1994, p. 158). Sherin (2001, 2006) indicates the presence of a boundary between formulas and concepts. This boundary is also visible in physics education literature. An important part of research in physics education literature stresses the importance of the naive knowledge structures (Clement, 1982; diSessa, 1993; Halloun & Hestesnes, 1985a, 1985b; McCloskey, Washburn, & Felch, 1983; McDermott, 1984), while another significant group of studies emphasizes the importance of problem solving in physics (Dufresne, Gerace, & Leonard, 1997; Huffman 1997; Leonard, Dufresne, & Mestre 1996; Mestre, Thaden-Koch, Dufresne, & Gerace, 2004; Reif, 1995; Reif, Larkin, & Brackett, 1976). However, Sherin (2001, p. 485) stated that "research into naive physics does not address solving textbook physics problems".

Traditional physic problems are often formula-centred and do not allow for the use of both intuitive knowledge and physics principles (Sherin, 2001). However, some scholars (Clement, 1994; Ploetzner & Spada, 1993; Ploetzner & Van Lehn, 1997) have raised the possibility of building a bridge between naive knowledge and problem solving, thus introducing the use of intuition into physics problem solving and acknowledging it as a part of the expertise in doing physics (Clement, 1994; Singh, 2002). It has also been suggested that intuition can be included in traditional physics problems in textbooks (Sherin, 2006). In the literature of physics education, this type of problem is named Elby pair question (Elby, 2001). Redish (2004, p. 45) defines Elby pair as "a pair of questions that ask the same physics question in two different ways. In one way, the context of the question cues a common student misconception with a high probability. In the second way, a different context cues a correct response".

Elby pair questions involve the exercise of a preference when solving a physics problem which makes it possible to determine the consistency or inconsistency in the results of students' problem solving. The preference shown by students when solving a problem can be associated with their epistemological belief. When exercising their preference, students will gain new knowledge or meaningful understanding based on their prior knowledge and, thus, they will able to develop strategies to solve the problem. Acquisition of new knowledge, meaningful understanding and strategies are integral to an epistemic game theoretical framework in the physics education literature (Tuminaro & Redish, 2007). The term 'epistemic' means construction of new knowledge to make sense of phenomena in the world. The term "game" means involvement of complex rules, strategies, and moves (Tuminaro & Redish, 2007).

Tuminaro and Redish (2007) developed the epistemic game theoretical framework to describe procedures used by students during problem solving in physics. They indicated that this framework is useful for investigating students' understanding and use of mathematics in the context of physics. Tuminaro and Redish (2007) defined six epistemic games in physics. In this study, I will focus on the following three epistemic games: (1) Physical Mechanism (PM) game, (2) Recursive Plug-and-Chug (RPC) game and (3) Mapping Mathematics to Meaning (MMM) game. The reason for selecting these 3 epistemic games is that they allow for an examination of whether students trust in mathematics or intuition when solving a particular physics problem. Solving a problem with intuitive knowledge and without mathematical calculation and explicit reference to physics principles may be regarded as Physical Mechanism (PM) game. In contrast, solving a problem by calculating only a concept using a physics

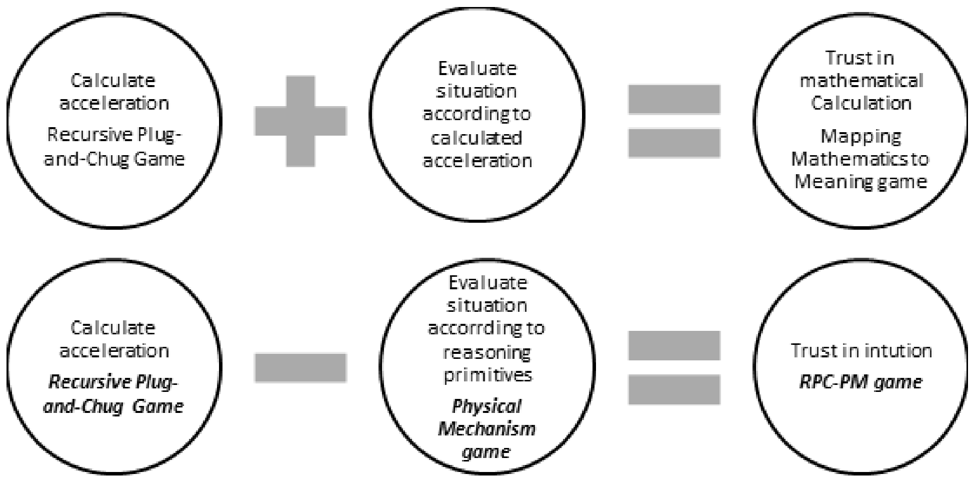

Figure 1. Trust in mathematical calculation and intuition according to epistemic game played

principle addresses Recursive Plug-and-Chug (RPC) game. Calculating the value of a concept, followed by evaluating and interpreting the result of the calculation corresponds to Mapping Mathematics to Meaning (MMM) game. Based on the three kinds of epistemic game, trust in mathematical calculations is to play MMM game. In contrast, trust in intuitive knowledge is playing both the RPC game and PM game (Figure 1). In figure 1, the plus (+) and minus (-) signs show whether or not students' race prediction are associated with acceleration in evaluation of an event in the problem. In this research, I focus on how students use epistemic games based on their epistemological beliefs while solving a particular physics problem.

Rationale of the study

The rationale for this study has two reasons. First, epistemic game theory is a recent theoretical framework developed by Tuminaro and Redish (2007) and is accepted as "useful in understanding how to teach strategies and metacognition in problem solving" (p. 1). Tuminaro and Redish indicated that "student's decisions (tacit or conscious) about which games to play have a critical role" (Tuminaro & Redish, 2007, p. 19) in problem solving. Similarly, Redish and Smith (2008, p. 300) stated that "the choice of e-game to play can be critical to students' success". Clearly, there is a need to investigate students' epistemic game preferences while they solve physics problems. As emphasized by Tuminaro and Redish, the MMM game represents an aspect of most intellectually complex epistemic games. Therefore, it is timely and valuable for physics educators to investigate students' trust in mathematical calculation in order to identify why and when students play the MMM game during problem solving.

Second, in the physics education literature, there have been several surveys including Maryland Physics Expectation [MPEX] (Redish, Saul, & Steinberg, 1998), and Colorado Learning Attitudes about Science Survey [CLASS] (Adams, Perkins, Podolefsky, Dubson, Finkelstein, & Wieman, 2006) to assess students' epistemological beliefs and expectations about physics problem solving. In these surveys, perceptions of students on the relationship between physics and mathematics were investigated. However, these surveys merely quantitatively have described students' trust in calculation during physics problem solving while solving a physics problem. It can be concluded that the existing surveys such as the MPEX and CLASS only assess students' epistemological beliefs about physics but they do not examine qualitatively the impact of trust in calculation during problem solving and the reasons behind the trust students have. To our knowledge, there is no study examining how trust in mathematical calculation and intuition influence coherence in problem solving. To this end, research questions guiding this research are as follows: (i) Do students trust in mathematics calculation to predict an event in a problem solving activity? (ii) What are the reasons behind their trusts? (iii) Is there a significant difference between trusts in mathematics and intuition and achievement in the General Physics I course?

Method

A mixed method approach (Creswell, 2009) was adopted in this research. Two qualitative investigations, a paper pencil session and interview sessions, were conducted. The paper-pencil session focused on determining students' epistemic game preferences, while the interview aimed at identifying reasons for trust in mathematical calculations and intuitive knowledge. A quantitative study was conducted to investigate whether there was a significant relation between students' trust in

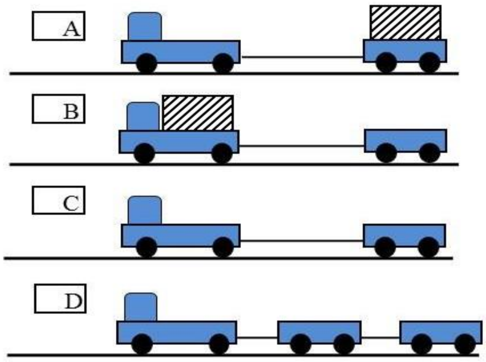

Figure 2. The drag-race set-up of the paper-pencil session

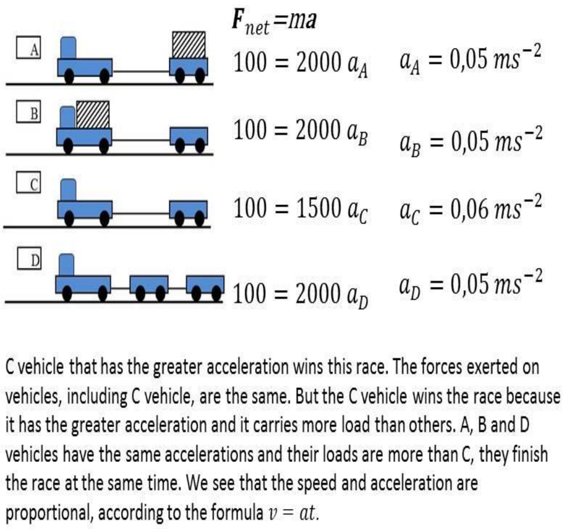

Figure 3. Reproduction of student B's solution

mathematics and achievement in the General Physics I course.

Paper-pencil session

Data collection process and participants

An Elby pair question based on race set-ups doing a drag race was used in the paper-pencil session (Figure 2). The first question asked students to calculate the acceleration of vehicles. The second question required them to predict which vehicle would win the race. Acceleration is a phenomenon not directly observable. It is well known that students tend to confuse acceleration concepts with velocity (Taşar, 2010). For this reason, the student participants were told that the initial velocity of the vehicles was zero and they raced with continuously increasing acceleration, in other words, a drag race. No suggestion such as 'predict the race result based on acceleration conclusions' or 'predict the race result based on own intuition' was implied in the question. Thus, students had the opportunity to choose which epistemic game to be played.

Trucks, trailers and loads were given as identical. The mass of the trailer and loads was equal (500kg) and smaller than the truck's mass (1000kg). The vehicle in situation C had the least mass (1500kg), and the total masses of the other set-ups were equal to each other (2000kg). It was indicated that the trucks provided equal pulling force (100N), and the truck and trailer were coupled with an inextensible rope with negligible mass. The initial velocity of the vehicles was zero and they uniformly accelerated in linear motion. When Fundamental Principle of Dynamics (FPD) was applied, the accelerations of vehicles A, B and D was calculated as equal and smaller than the acceleration of the vehicle C. Thus, it could be concluded that the vehicle with the greater acceleration would have higher velocity, applying the equation "v=at", and it could be predicted which vehicle would win the race.

The question set was administered in the 2012-2013 academic year to a group of 83 first year students who took the General Physics I course in a public university. In Turkey, students admitted to university have taken the Central National University Entrance Examination (CNUEE). Therefore, it could be established that the participants of this study had similar CNUEE scores though they came from different socioeconomic levels. At the university, they studied Newton's Laws in the General Physics I course, and so were equipped with the knowledge to respond the questions about the Newton's Laws. No time limit was given to complete the question set. It was observed that students completed the question set within 10 minutes.

Data Analysis Process

First, students' solutions were classified under two main categories as shown in Table 1. The first category was Mapping Mathematics to Meaning (MMM) game, which involved calculating the acceleration of the vehicles and then predicting race ranking by comparing their accelerations. The second category was RPC-PM game, which involved calculating the acceleration of vehicles and predicting race ranking without reference to the acceleration calculations. In the second category the acceleration calculations was the Recursive Plug-and-Chug (RPC) game while prediction of race ranking was is the Physical Mechanism (PM) game. In the second stage of the analysis, correctness of solutions was evaluated. Student' solutions were coded in six separate categories by the author. These categories contained all the possibilities relating to the characteristics of the epistemic game played and the correctness of the solutions (Table 1). During analyses, the solution corresponding to the 6th Category was not found. So, solutions were coded based on five categories. Coding was done by two physics educators. The inter-coder reliability was 95% according to Miles and Huberman's (1999) formula: Reliability = (number of agreements)/(total number of agreements + disagreements). According to Miles and Huberman reliability should be greater than 70%.

MMM game Category. There are four moves in the MMM game according to Tuminaro and Redish (2007).The first move is to identify the target concept. The target concept is the concept of acceleration in the problem. The second move is to find an equation relating the target concept to other concepts. The third move is to develop a story between the concepts such as "greater acceleration, greater velocity". The final move is to evaluate the story. The solution given by Student B is reproduced in Figure 3 as a sample analysis.

Student B properly used FPD and correctly calculated the acceleration of the vehicles. Vehicle C had the higher acceleration while the acceleration of the others were equal and smaller than C. The calculations corresponded to Moves 1 and 2 of the MMM game. Student B determined the race winner according to the acceleration calculation. She developed a story by indicating that the vehicle with higher acceleration would be faster. She also justified this story with the linear motion equation v=at. This solution corresponds to Category 1 in Table 1. It can be concluded that Student B trusts mathematical calculation.

RPC-PM Category. To exemplify this category, Student O's solution has been reproduced in Figure 4.

According to Tuminaro and Redish (2007), RPC game consists of 3 moves: (1) identifying the target concept (acceleration), (2) finding an equation relating the target to other concepts (FPD) and (3) calculating the target quantity (calculation of acceleration value). The acceleration calculation of Student O corresponds to these moves. He found the accelerations of Vehicles A, B and D to be equal and smaller than the acceleration of vehicle C. He indicated that vehicle C would be the winner because of its smallest load. However, he did not refer to the acceleration calculation. Similarly, he made no reference to the calculation when predicting the winner. An examination of Student O's race prediction showed that he compared primarily the loads. Although it was given that the loads were are equal, he considered the load distribution. Then he applied the criterion of load proximity to the engine. Student O's explanation corresponds to moves of the PM game: "developing story about physical situation" (Move 1) and "evaluating story" (Move 2). Student O considered acceleration calculation and race prediction as two separate tasks. He did not associate the acceleration results with the race prediction. Hence, his solution was coded as RPC-PM game. Although the FPD application and acceleration results were correct, the intuitive race prediction was incorrect. Student O's solution corresponds to Category 4 in Table 1.

Interview sessions

Four students from each category were selected for interview based on the paper-pencil analysis and a total of 20 students were invited to the interview. The students consented to participate in the interview. Interviews were conducted two months after the paper-pencil session. The reason for the delay was to prevent cross-pollination of ideas among students after the paper-pencil session and avoid potential misleading input which discussion with friends might have caused. The time gap between the two data collection procedures was justified by the identical results obtained in interviews and the paper-pencil session even though the students had been subjected to a time lapse between the two investigative methods. The interviews took approximately 10-15 minutes each on average and students were encouraged to express their thoughts freely.

The interviews were semi-structured, consisting of

Table 1. Codes used for Categories of students' solutions in paper-pencil session

Solution categories	Epistemic game(s)	Features of categories
1	MMM game	Acceleration calculation correct — race prediction according to correct acceleration calculation
2		Acceleration calculation incorrect — race prediction according to incorrect acceleration calculation
3	RPC-PM game	Acceleration calculation incorrect— Incorrect intuitive race prediction
4		Acceleration calculation correct — Incorrect intuitive race prediction
5		Acceleration calculation correct — Correct intuitive race prediction
6		Acceleration calculation incorrect— Incorrect intuitive race prediction

three main sections: acceleration calculations, comparison of acceleration results, and race prediction. The following probe questions were posed to the students:

1. *Which of these set-ups would win the race?*
2. *How would you rank the vehicles' chances of winning? Can you explain your ranking with reasons?*
3. *How do you calculate the accelerations of the set-ups?*
4. *Do you compare the accelerations of these set-ups?*
5. *Is there any inconsistency between the ranking of accelerations and the race result? If yes, what are the reasons?*
6. *Do you think that the difference between the acceleration comparison and the race result is important?*
7. *Which do you think is more logical? The acceleration calculation result or your prediction?*
8. *If we were to give you another chance to do this exercise, which method would you prefer? Your prediction or the acceleration comparison?*

Questions 1-3 are paper-pencil session questions, designed to recall the set-ups and to compare students' solutions in the paper-pencil session with those given in the interview. Questions 4-7 were asked to discover whether students associate acceleration calculation with the race result. Question 8 was asked to determine whether students have trust in mathematic calculation or intuition.

In the interviews, the researcher took care not to impose any strategy on the students. During the interviews, it was observed that some students quickly calculated acceleration when the interviewer asked which vehicle would win the race. Later, they were asked why they preferred calculations as the first course of action instead of responding intuitively. They were also asked probing questions in order to uncover their intuitive responses. In the case of students who showed reliance on mathematics rather than their intuition, the following question was asked: "If the set-ups A, B, C, and D accelerations were calculated respectively as 0,3; 0,2; 0,1 and 0,2 m/s, what is your race ranking?" These values imply that the set-up with the least mass would arrive slowest, a response that is intuitively not logical and quite counterintuitive. Thus, it was possible to identify to what extent students were reliant on calculation or intuition.

Quantitative Analysis of Students' achievement and epistemic game preferences

The student participants attended General Physics I in which the researcher was the instructor. An independent t test was conducted to compare the means of the students' scores in General Physics I and their performance in both MMM game and RPC game categories. A one-way ANOVA was performed to determine if students' achievement was related to the solution categories in Table 1. Finally, a Scheffe test was performed to examine differences between the categories.

FINDINGS

Finding 1. Students' epistemic games preferences

Students' epistemic game preferences were analysed based on the criteria listed in Table 1. The results can be seen in Table 2 which show that MMM game players form 43% of the total while RPC-PM game players made up 57%. The higher percentage of RPC-PM game players suggests that the majority of the students viewed acceleration calculation and race result prediction as two unrelated tasks.

49 % of the students (in categories 3 and 4) produced incorrect intuitive race results arising from factors related to the mass of vehicles (e.g., load on the truck or on the trailer). Forty four per cent of students (in categories 2 and 3) incorrectly calculated the acceleration of vehicles. Their difficulties stem from problems in identifying and selecting a system for the application of FPD to the multi component systems. Some students focused on the truck instead of considering the set-up as a whole system and ignored the effect of the trailer. As a result of these errors they were unable to calculate the expected acceleration value.

Category 2 and Category 4 respectively show absolute trust in mathematical calculation and intuitive knowledge. Students of Category 2 incorrectly calculated the acceleration of vehicles and predicted the race result according to this incorrect acceleration calculation. Unquestioned trust in mathematical calculation can lead to a counter intuitive race prediction, for example, that vehicle C with the least mass failed to finish the race as the winner. This is an example of absolute trust in mathematical calculation. Students of Category 4 correctly calculated the acceleration of the vehicles. However they predicted a race ranking according to intuitive knowledge with no regard for the calculations. This is example of absolute trust in intuition.

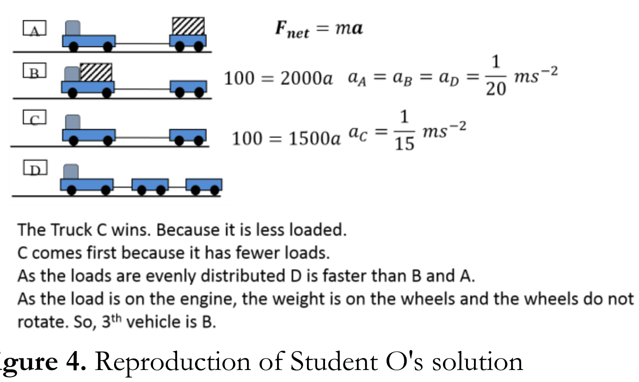

Figure 4. Reproduction of Student O's solution

Finding 2. Math does not lie!

Students in Categories 1 and Category 2 did race prediction according to acceleration results and they trusted in mathematical calculation. Students who adhered to axiom "Math does not lie!" indicated that their trust in mathematical calculations was based on two reasons: (1) mathematics is more convincing and realistic than verbal intuitive explanations, and (2) intuitive knowledge leads to incorrect conclusions in physics. During the interviews, it was observed that some students preferred mathematical calculations even if these calculations obviously conflicted with intuition. These students argued their position based on their experiences and perceptions of physics problems. Therefore, it can be seen that correct or incorrect calculation of acceleration and conflict with intuitive expectations seem not to be important for students who played the MMM game. In order to determine the extent to which students trust in mathematical calculation, interviews with Student B and Student F are analysed below. Student B revealed trust in the persuasive nature of mathematical calculation, while Student F highlighted a surprising aspect of physics.

Example 1. Plausibility in Mathematical Calculations

The analysis of the paper-pencil solution of Student B (Category 1) has been presented in Figure 2. In the interview, she did not respond immediately to the question "which will win race?". She did not give a race prediction without calculating the acceleration. She first calculated the accelerations and then did the race ranking based on the acceleration results. She was asked why no response was given immediately and why the race ranking was done by referring to the acceleration results using FPD. Her answer was:

"Well, I first wanted to apply a formula. Then I thought it would be solved with F=ma. I thought it would be more

Table 2. Distribution of students' solutions

Solution Category	Played Epistemic Game	Explanations		Number of students who played epistemic game f	%	Total f	Total %
1	MMM game	Trust in mathematical calculation	-Correct acceleration calculation -Correct race prediction: race ranking according to correct acceleration results	18	22	36	43
2			-Incorrect acceleration calculation -Incorrect race prediction: race ranking according to incorrect acceleration results	18	22		
3	RPC-PM game	Trust in intuitive knowledge	-Incorrect acceleration calculation -Incorrect race prediction: intuitive prediction independent of acceleration and in conflict with calculation	18	22	47	57
4			-Correct acceleration calculation -Incorrect race prediction: intuitive prediction independent of acceleration and in conflict with calculation	22	27		
5			-Correct acceleration calculation -Correct race prediction: intuitive prediction independent of acceleration	7	7		
Total						83	100

reliable. One could know at first look. For example, C does not have load and [the empty truck] is pulling a single trailer. The others, for example A and B [set-ups] could arrive at the same time. First I thought of these. But, to make it a more correct answer, I thought that using a formula was more logical."

These explanations clearly show that Student B was engaged in the MMM game. She expressed an intuitive approach as "knowing at first look". But she felt the need to verify the intuitive race prediction with the sentence"… a more correct answer..." Thus, she provided evidence of relying on the use of mathematics as a verification of intuitive knowledge. The interviewer showed Student B her paper-pencil solution and asked why she was not satisfied with the intuitive race prediction. The dialogue between Student B and the interviewer (I) proceeded as follows:

I: Why did you answer here by comparing the accelerations?

Student B: Sir, you know why I did not answer like that [intuitively]? I thought answering simply with a look at the figure would be too short. Saying only there is no load on "C" and it will be first seemed too short to me. Generally I answer questions with proof. Proof is more logical for me. Finding numerically first and support of both is more logical.

I: OK, what would you have done if you hadn't found the proof?

Student B: Numerically?

I: Yes.

Student B: Hmmm...

I: For example, if you hadn't found what you had in mind, what would you have done numerically? For instance, you've made calculations and discovered that "C" finishes last. But, on the other hand you're thinking "C" should've been first. What would you have done, would you have relied on the calculation? Or...

Student B: Personally, I would've trusted numerical more.

From Student B's explanation, three points may be noted. (1) According to Student B, the presence of mathematical operations and calculations in problem solving contributes weight, persuasiveness and plausibility to the solution. She considered an answer without mathematical calculation as "too short". She said "Saying only there is no load on "C" and it will be first seemed too short to me." (2) Trust in mathematical calculations is related to a student's physics problem solving style. This observation is evident from her explanation: "Generally I answer questions with proof. Proof is more logical for me. Finding numerically first and support of both is more logical". (3) She expressed greater trust in mathematics than in intuitive expectation: "Personally, I would've trusted numerical more". This shows that for this student "math does not lie!" despite the absence of alignment between the mathematical results and the intuitive expectation.

Example 2. A surprising aspect of physics: Interview with Student F

Student F (Category 2) made the race prediction using incorrect acceleration results. She asserted in both paper-pencil and interview sessions that the vehicle with the least mass would lose. This prediction is contrary to intuitive expectation. In the interview, she said, "For me C being last was really illogical. But according to the calculations I've done here that's how it was!". The fact that the quantitative results did do not coincide with intuition did not shake Student F's trust in mathematics. When the interviewer ask the question "even if not logical?", Student F replied, "Mathematical calculations are more important than visuality". By the term "visuality", she was referring to the intuitive connotations created by the set-up configuration. We may conclude from Student F's explanations that (1) she was aware of a conflict between the acceleration calculation and her intuitive race prediction; and (2) her preference in problem solving was for mathematical calculation.

Days after the interview with Student F, she revisited the interviewer to explain her thoughts relating to "visuality and mathematics". In the short conversation, Student F explained that intuitive expectations in physics would not always lead to correct results:

"Whatever I see turns out wrong. I think of calculations as always correct. For instance consider an elephant and a piece of chalk. In vacuum someone who doesn't know [free fall motion in physics] might say directly this one [the elephant] would fall. But their fall will be same. A person might think that the heavier one will fall first but in vacuum both will fall at the same time. One needs to know. I always say formulas are correct. Sir, physics always surprise me!"

Student F pointed out a dichotomy: seeing and knowing. While seeing involves perception and common sense dimensions about an event, knowing brings into play the theoretical and conceptual dimensions of scientific knowledge. The following words of Student F show her awareness of the unreliability of perception and common sense in seeing in her 'elephant and chalk' analogy: 'whatever I see turns out wrong', 'someone who doesn't know might say this would fall', 'person might think the heavier one will fall first'. The conceptual dimension in her analogy is related to physics laws and manifested in her assertion about the reliability of calculations and formulas: "I think of calculations as always correct", "One needs to know. I always say formulas are correct".

Finding 3. Intuition does not mislead!

In the paper-pencil session, 47 of the 83 students (57%) regarded "calculate acceleration" and "predict race result" as two separate tasks (see Table 2). These students played two epistemic games: RPC and PM. Analyses of the interviews indicate that (1) students who played the RPC-PM game did not perceive any irregularity or conflict in their solutions, and (2) their trust in intuitive knowledge prevented them from playing the MMM game. It appears that acceleration calculations influenced predictions of the race result but the calculations were not the only resource relied on because intuitive knowledge are taken into consideration. As a result, trust in intuitive knowledge among the students who played the RPC-PM game was found to be higher than confidence in mathematical conclusions. This finding is exemplified below in a close examination of the interview responses of Students O and T, which show to what extent students do not trust mathematical calculations.

Example 1. Acceleration is important but …

In the paper-pencil session, Student O (Category 4) correctly used FPD and accurately calculated the vehicles' accelerations. His solution is analysed in Figure 4. Student O disregarded his acceleration calculations, and intuitively predicted an incorrect race ranking. Since the concept of acceleration was not associated with the race result, Student O played RPC-PM games in arriving at his solution.

In the interview with Student O, the interviewer showed the same set-up and asked him to predict the race ranking. Student O provided the same answer he had previously given in the paper-pencil session (Ranking in Figure 3, C>D>B>A). The interviewer drew his attention to the relationship between the pattern of his acceleration calculations (aC>aA=aB=aD) and his race ranking prediction. The interviewer asked whether there was a problem in the relation between the two answers. It was observed that Student O remained unaware of any conflict or problem in his solution. He even asked the interviewer if the problem was due to the acceleration calculations. As soon as he perceived a conflict in the solution he asked the interviewer not take into account the calculations but only to consider the intuitive prediction. Student O's strategy of dealing with the conflict in his solution shows the extent of his trust in intuitive knowledge. An excerpt from the interview with Student O is given below.

I: Compare the accelerations and race rankings. Which should we prefer? Do you think there is a problem here?
Student O:...
I: Do you understand?
Student O: Sir, I didn't grasp it.
I: [Interviewer summarizes the question] you've calculated the acceleration. The highest acceleration is C and the accelerations of the others are equal. Now, if I look at the accelerations, D, B and A are equal. If I look at the ranking....
Student O: Problem because of acceleration? What? Sir, it appears I haven't done it according to acceleration?
I: Well, if that's the case, what use are accelerations?
Student O: Sir, please look at race ranking; don't look at acceleration!

From Student O's interview responses, it can be observed that his intuitive expectation caused him to disregard the acceleration calculations. He had declarative knowledge of the meaning of acceleration. For example, he could say that the velocities of vehicles increase in the same way if their accelerations are equal. He also predicted that Vehicle C would will be the winner. However, he failed to take into account the equality of the accelerations of Vehicles A, B, and D. Twice during the interview he said that the mass of the load did not matter when the interviewer called his attention to the equal acceleration of the vehicles. At the end of the dialogue, the interviewer gave Student O a final choice, encouraging him to produce a race ranking according to the acceleration results. However, Student O had difficulty again in assigning a mean to the acceleration calculation results and preferred intuitive race ranking. We may conclude from the interview excerpt below that for Student O intuitive expectation is more important than mathematical calculation results.

I: In that case what does equality of accelerations mean?
Student O: Sir, it means their velocities are increasing the same way. For instance, if C had higher acceleration it would have come first.
I: Yes, C is the winner. But, accelerations of A, B and D are equal.
Student O: They need to be head to head… but... Aren't loads important?
I: Are you saying accelerations are not important?
Student O: Sir, I want to ask something. Does load have no meaning then? For instance load at front or at rear, make any difference? Sir, now, according to part (a), C should be first and others second. But, Sir, I am not sure, does not location of load affect velocity?
I: If I were to ask you the final time, which is correct?
Student O: Now, if I were to rank according to acceleration it will be C>A=B=D. These [A, B and D vehicles] should finish at the same time. Then, I am disproving my answer in part (c).
I: In that case, load is not important for you?
Student O: But, Sir, shouldn't the location of load be important? Sir, I prefer the other ranking [intuitive race ranking].

Example 2. What about my logic?

The second example is an excerpt from the interview with Student T (Category 3). Student T intuitively predicted the race result before calculating the acceleration of vehicles in her paper-pencil solution. In the interview session, she used the same method of solving the problem. She first predicted the race winner and then calculated the vehicle accelerations. This sequence in her problem solving procedures clearly shows that Student T like other students of Category 3 and 4 regarded acceleration calculation and race result prediction as two separate tasks and attached great importance to intuition rather than calculations.

I: Which will win the race?

Student T: C [vehicle] would be the fastest. Since its load and friction are less than others. Later D will arrive. Then B. Because its load is on the trailer behind and tension force on the rope is higher than others. As the cable tension force increases the pull force will decrease.

Student T made no reference to physics equations, indicating the PM game. The story developed by Student T can be summarized as "loaded vehicle with load on the trailer goes slower". She explained this story by using the notion of friction force. At the interview she was first asked to rank the race and then calculate the accelerations of the set-ups. She repeated the ranking stated in the excerpt above and calculated the accelerations correctly.

When asked to give a race prediction based on the accelerations, she indicated that the C set-up would win the race. For a moment or so she entertained the idea that the equal accelerations of the other set-ups would result in a head-to-head race finish. But she later abandoned this idea and returned to intuition, explaining her decision thus: "All are equal, all will be second... May be may be not... But where the loads are and number of trailers pulled, not important?". Student T was aware of the conflict in her set-up rankings, and when asked her preference, she chose the intuitive solution. The dialogue below reveals that although she was temporarily indecisive, she preferred intuition. Her belief in the falsifiability of mathematical physics principles is highly significant and may explain her absolute confidence in intuition.

I: Decide then, which would you have presented to me as an answer?

Student T: This one! [Student points at intuitive conclusion.]

I: Fine, why that? What is the reason?

Student T: When we think logically, if I view the figure according to what I have in my mind...

I: Did your logic not comprehend the acceleration results ranking?

Student T: In reality that too... No, I would've taken according to this [intuitive result]. I say the formulas can be disproved. There is no such thing like always correct just because formula is proven! Not like absolutely can't be disproven logic. There is no such thing as it will certainly be correct with all inputs! Personally, I won't remain loyal to the formula!

Table 3. Students' mean scores in General Physics I by solution categories

Solution categories	f	Mean	Std. Deviation
1	18	50,1222	11,10557
2	18	41,9278	10,11119
3	18	36,3722	11,38755
4	22	36,1455	8,40106
5	7	42,2857	12,55912
Total	83	40,9976	11,52759

Table 4. Independent Sample t test results

Epistemic games	N	Mean	Std. Deviation	t
MMM game	36	46,0250	11,26185	3.743*
RPG-PM game	47	37,1468	10,26892	

**p< 0,05*

Table 5. One-way ANOVA results

	Sum of Squares	df	Mean Square	F
Between Categories	2428,893	4	607,223	5,593*
Within Categories	8467,706	78	108,560	
Total	10896,600	82		

**p< 0,05*

Finding 4. Trust in mathematics or intuition and students' achievement

Physics achievement scores of students placed in the five categories are presented in Table 3, which shows that while more successful students were found in Category 1, less successful students were found in Categories 3 and 4.

An independent sample t-test was performed to ascertain if there is a significant difference between the mean scores of students in the MMM and RPC-PM categories. The result can be viewed in Table 4 which shows that students who played the MMM game are significantly more successful in physics than students who played the RPC-PM game. It appears that students who trust in mathematics are more successful in physics. Such a finding may be expected because of the calculus based nature of general physics courses. This finding reinforces the idea expressed by some students in the interviews: "to succeed in physics one must believe in maths". This idea was expressed by interviewees in the MMM game category.

A one-way variance analysis (ANOVA) was performed to discover the pattern of relationship between students' achievement in General Physics I and the categories of MMM game and RPC-PM game. The one-way ANOVA results are given in Table 5 which shows significant differences between the categories and mean scores within the categories. The Scheffe test was performed to discover the location of significant differences between categories. The results are in Table 6.

Results of the Scheffe test in Table 6 show a significant difference between Category 1 and Category 3, and between Category 1 and Category 4, with the highest physics scores occurring in Category 1. Students in Category 1 correctly calculated accelerations and predicted race results according to these accelerations. Students in Category 3 incorrectly calculated accelerations and predicted the race result using primitive reasoning. Students in Category 4 correctly calculated accelerations but did not apply the calculations to prediction of the race result. Thus, the significant difference between Catgories1 and 4 appears to be more important. Although students in Categories 1 and 4 correctly solved the problem they did not have the same level of success in General Physic I. This finding suggests that playing the Mapping Mathematics to Meaning game is more effective in physics than playing correctly the Recursive Plug-and-Chug game.

Table 6. Results of Scheffe test relative to significance of differences

(I) Group	(J) Group	Mean Difference (I-J)	Sig.
1	4	13,97677*	,003
	5	7,83651	,586
	2	8,19444	,245
	3	13,75000*	,006
4	1	-13,97677*	,003
	5	-6,14026	,764
	2	-5,78232	,553
	3	-,22677	1,000
5	1	-7,83651	,586
	4	6,14026	,764
	2	,35794	1,000
	3	5,91349	,804
2	1	-8,19444	,245
	4	5,78232	,553
	5	-,35794	1,000
	3	5,55556	,636
3	1	-13,75000*	,006
	4	,22677	1,000
	5	-5,91349	,804
	2	-5,55556	,636

DISCUSSIONS AND CONCLUSION

In this study I investigated the extent to which students place their trust in intuitive knowledge and mathematical calculation. The purpose of this study was to seek answers to the following questions: (i) Do students trust in mathematics calculation when predicting an event in a problem solving activity? (ii) What are the reasons for their trust in mathematics or intuition? and (iii) Is there a significant difference between trust in mathematics or intuition and achievement in the General Physics I course? An Elby pair question was used to investigate participants' problem solving preferences.

In this research trust in mathematical calculation was defined as relying on acceleration calculations to predict a race result. Trust in intuition was defined as relying on primitive reasoning to predict a race result without referring to acceleration calculations. Trust in mathematics or intuitive knowledge was conceptualized within a recent theoretical framework known as epistemic game. Students' preference for mathematical calculation was viewed as playing the epistemic game of Mapping Mathematics to Meaning (MMM). Preference for intuition was viewed as playing two distinct games: the Recursive Plug-and-Chug game (for calculating acceleration) and the Physical Mechanism game (for predicting race results).

Results from the paper-pencil session revealed that 43% of the students predicted race results on the basis of calculated accelerations, while 57% predicted race results according to primitive reasoning without consulting calculated accelerations. This first finding shows that almost half of the 83 students trust in mathematical calculation while the other half trust in intuitive knowledge. From this finding, we may conclude that 57% of the students considered acceleration calculation and race result prediction as two unrelated tasks. It was also found that some students correctly calculated the accelerations but made no reference to them subsequently, suggesting a lack of understanding of the concept in the winner prediction task. On the other hand, it was found that some students could not correctly calculate acceleration but nevertheless they relied on their acceleration calculation, which then led them to prediction results counter to their intuition. This finding confirms the view that students' epistemological beliefs have an influence on the cognitive processes of thinking and reasoning (Hofer & Pintrich, 1997) and on problem-solving performance (Jonassen, 2000).

Another finding from this research is that students who played the MMM game had a strong belief that "Math does not lie!". These students indicated that their trust in mathematical calculations stems from two reasons. The first is a belief that mathematics is more convincing and realistic than verbal intuitive explanations. The second is their distrust of intuition arising from their view that intuition, common sense and intuitive expectation lead to incorrect conclusions in physics. These students place their trust in mathematical calculation. After the interview sessions, one student returned to the interviewer for additional explanations. During their discussion, she reported that "our physics teachers said if you want to answer physics questions successfully in the university examination, never trust your feelings. Just apply the physics equations and consider only the results. The teachers used to say we should not let intuition interfere with how we think or allow intuition to question the results, why the results are this way or that way." This explanation of the student suggests that one must believe in math to be successful in physics. There is absolute trust in mathematics. This perspective seems appropriate for students interested only in short-term success. However, it deprives them of the option of questioning the result of mathematical calculations. It reinforces the idea that solving physics problems is just a matter of calculating physical quantities.

The interviews conducted with the students who considered acceleration calculation and race prediction as two distinct questions revealed their belief that intuition does not mislead. These students played two distinct games, the Recursive Plug-and-Chug and the Physical Mechanism game. Thus, calculations were done accordingly on the principles of physics, and intuitive knowledge was the outcome of primitive reasoning. It was observed that for these students there was no need to see a relation between the two tasks. Hence, they did not conceive any conflict in their solutions. For them, calculation and prediction were separate tasks. It was also observed that their trust in intuitive knowledge prevented them from playing the MMM (Mapping Mathematics to Meaning) game. When students in this category felt themselves forced to use the MMM game, they were not able to end the game. There was an absence of the beginning and ending condition, which is a structural component of epistemic games.

Students' achievement in General Physics I and their epistemic game preferences were compared to determine if there is a significant difference between the two variables. It was observed that students who played the Mapping Mathematics to Meaning game had higher mean scores in General Physics I than students who played the Recursive Plug-and-Chug and Physical Mechanism games. The Scheffe test revealed a significant difference between students who predicted the race result based on correct acceleration calculation and students who intuitively predicted the race result and incorrectly/correctly calculated acceleration. It can be concluded that playing the Mapping Mathematics to Meaning, one of most intellectually complex epistemic games, is a factor in achievement in physics.

The reason for absolute trust in mathematical calculation in physics can be traced to the historical development of Newtonian mechanics and its epistemology. Although Newton's "Philosophiae Naturalis Principia Mathematica" was published in 1687, mathematization of Newton mechanics did not occur at the time of Newton and the century following (Blay, 1995; Yavuz, 2007). Until Lagrange's "Mécanique Analytique" in 1788, there was no widespread use of mathematics; there were geometric proofs and experimental validation of results. Lagrange (1788) announced in the preface of his book that he reduced problem solving methods to general formulas by means of which solutions might be obtained easily. In the period following Lagrange's publication, trust in mathematics increased, gradually displacing validation through experiments. As a result, in today's physics education literature, the approach to arriving at solutions is known as the formula centred approach (Champagne, Gunstone, & Klopfer, 1982; Chi, Feltovich, & Glaser, 1981; Heller & Reif, 1984), with absolute and unquestionable acceptance of mathematics viewed as the norm.

Inability to trust in mathematical calculation can be explained by Hammer's (1994) epistemic beliefs framework. The second dimension of Hammer's framework is content knowledge, which includes formulas centred or conceptual. He developed the idea of apparent concept, which is present when "One can check the result of a calculation against one's informal knowledge of velocities without thinking of the calculation itself as conceptually accessible." (Hammer, 1994, p. 162). In the current study, for the students who trusted in mathematical calculation acceleration was an apparent concept. In contrast, for the students who trusted in intuitive knowledge, acceleration was not an apparent concept as they had difficulty understanding the implications of the acceleration calculation results in the race prediction task. The students' difficulty in constructing acceleration as an apparent concept mirrors the results of other studies on the learning of the acceleration concept (Coelho, 2010; Taşar, 2010; Trowbridge & McDermott, 1981).

Sherin (2011) suggested that quantitative problem solving and intuitive knowledge can be used together with intuitive knowledge forming a context for interpretation of quantitative results. The findings in this study appear to confirm the fact that epistemological beliefs can prevent the correct use of intuitive knowledge in problem solving. It was found that some students showed absolute trust in mathematical calculation regardless of intuitive expectations while others trusted in intuition regardless of the mathematical results. This finding raises an important question on the role of traditional problem solving exercises in introductory physics courses. Traditional physics problems generally focus on quantitative solutions and do not include intuitive reasoning. Consequently, some students in Category 3 and Category 4 were not even aware that the two questions (acceleration calculation and race result prediction) were related by a single purpose. The problem posed to the students in the current study also shows that Elby pair type questions (Elby, 2001) can enhance traditional physics problems by adding a test of intuitive expectations to the test of the application of physics principles.

In future studies, students' trust in mathematical calculation or intuition in other branches of physics should be investigated. In addition to the Newtonian motion laws, one line of research could pose a set of questions involving magnetism, optics and modern physics topics including the application of physics in situations directly associated or not associated with daily life. Another line of research could be focused on the improvement of epistemological beliefs about mathematics and intuitive knowledge in general physics courses.

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