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Costa Rican students' proportional reasoning and comparing probabilities in spinners

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Abstract

This research aimed to relate Costa Rican students (11-16-year-olds) competence to compare probabilities in spinners and proportional reasoning in the comparison of ratios. We gave one of two questionnaires to a sample of 292 students (grade 6 to grade 10) with three probability comparison and three ratio comparison problems each. Globally both questionnaires cover six different proportional reasoning levels for each type of problem. Additionally, each questionnaire contains two comparison probabilities items intended to discover a specific bias. We analyze the percentages of correct responses to the items, strategies used to compare probabilities per school grade, and students' probabilistic reasoning level. The results confirm more difficulty in comparing ratio than in comparing probability and suggest that the reasoning level achieved is lower than established in previous research. The main bias in the students' responses was to consider the physical distribution of colored sectors in the spinners. Equiprobability and outcome approach were very scarce.

Keywords: comparing probabilities in spinners, proportional reasoning, reasoning levels, 11-16-year-olds students

INTRODUCTION

Probability is part of the mathematics curricula in primary and secondary education in Costa Rica (Ministerio de Educación Pública [Ministry of Public Education] [MEP], 2012) and other countries (e.g., Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020, Ministerio de Educación y Formación Profesional [Ministry of Education and Professional Training] [MEFP], 2022). The reasons include usefulness of topic in decision-making, its instrumental role in other subjects and the study of inference, and the need to understand the probabilistic information prevalent in the media (Borovcnik, 2016; Gal, 2005; Muñiz-Rodríguez et al., 2020; Vásquez et al., 2021).

Piaget and Inhelder (1951) first studied the problem of comparing two probabilities, and afterwards, many researchers investigated the children's capacity for this task (Batanero & Álvarez-Arroyo, 2023; Hernández-Solís et al., 2023; Jones et al., 2007; Pratt & Kazak, 2018). Children participating in these studies had not previously studied probability and revealed a strong relationship between success in probability comparison and proportional reasoning. Moreover, proportional reasoning is considered to be linked to the acquisition of probability reasoning (Begolli et al., 2021; Bryant & Nunez, 2012; Watson & Shaughnessy, 2004).

Most research analyzing the comparison of two probabilities used the selection of balls from urns, and few of them considered at the same time the proportional reasoning of participants. In a related paper (Batanero et al., in press) we analyzed the responses of 704 11-16-year-olds students from Costa Rica and Spain in the comparison of ratios and the comparison of probabilities in urns. The tasks covered six different proportional reasoning levels for each type of problem. The results confirmed the highest difficulty in comparing probability that in the comparison of ratios and that the reasoning level achieved in both tasks were lower than suggested in previous research. We also informed of correct and incorrect strategies of students in the comparison of probabilities.

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Contribution to the literature

- This study adds to the literature new results related to students' performance in comparison of probabilities in Spinners and proportional reasoning. We compare correctness of response and strategy used in both types of problems in 11-16-year-olds.
- Another novelty is the comparison of responses in biased and non-biased items that revealed more incorrect strategies in the latter.
- The questionnaires used and categories of analysis that may help teachers evaluate the proportional and probabilistic reasoning of their students.

This paper complements this research by evaluating the comparison of probabilities in the context of spinners and their relationship with the proportional reasoning level in a sample of Costa Rican students from 11 to 16 years of age. Additionally, we analyze the possible existence of reasoning biases due to the distribution of favorable sectors in the spinners. Specifically, we pose the following research questions:

1. Which percentage of students in each grade correctly compare probabilities in spinners in tasks corresponding to the different proportionality reasoning levels of Noelting (1980)? Do these levels coincide in the comparison of ratios?
2. What are common correct and incorrect strategies when comparing probabilities in spinners by grade and proportionality reasoning level? Do some strategies change in the biased items?
3. Which probabilistic and proportionality reasoning level do students reach in each grade? Are these levels related?

The article is structured, as follows: Firstly, we describe the theoretical framework and background, continuing with the methodology and the presentation of results. The paper concludes with a discussion of results and implications to improve the students' probabilistic and proportional reasoning.

FOUNDATIONS

Early Study by Piaget and Inhelder (1951)

Piaget and Inhelder (1951) conducted the first studies on probability with children aged 3.5 to 14. They used a few cards, marked or not with a cross. In personal interviews with the children, they asked them to select the most likely group to obtain a marked card between two groups of such cards. The researchers varied the number of marked and blank cards in the groups, and, after many such interviews, they deduced a series of reasoning stages in the probabilistic reasoning of children.

Piaget and Inhelder (1951) divided the first stage (I) in three: children in level IA do not understand the inclusion of a part in a whole, the disjunction between two types of elements or the conservation of quantities. At level IB, children compare only one kind of event (favorable or unfavorable) and do not conceive the favorable cases as part of the possible cases (part-whole comparison). At stage IC they can compare two probabilities in three cases:

(a) double impossibility,

(b) double certainty, and

(c) certainty-impossibility.

They divided the second stage (II) into two: at level IIA, children use additive comparisons (e.g., subtracting the number of favorable cases from the number of unfavorable events, or vice versa, in each urn and comparing the differences). At level IIB, children start solving the problem when the composition of the groups is proportional. To do so, they establish a correspondence between the favorable and unfavorable cases in one group and compare it with that in the other group.

At stage III, the child can solve the proportional case and think of a general strategy if the ratios between the favorable and possible events are simple. This solution becomes more general with age as the child acquires sufficient knowledge of fractions.

Proportional Reasoning & Strategies in Comparing Ratios

The wide research on proportional reasoning has focused on the different meanings of the rational number (Burgos & Godino, 2020). This research is compiled, among other papers, by Ben-Chaim et al. (2012), Carpenter et al. (2012), Lamon (2007), Kieren (2020), Obando et al. (2014), and Van Dooren et al. (2018).

The comparison of ratios served to study the development of proportional reasoning and expand the levels proposed by Piaget and Inhelder (1951) in probabilities. In his research, Noelting (1980a, 1980b) used the problem of mixing water and orange juice in different proportions and asked the children, which mixture was stronger flavored.

Noelting (1980a, 1980b) described each mixture as a pair (a, b), where the first term was the number of glasses of orange juice (a), and the second was the number of glasses of water (b). In this research, we use his classification, as follows:

1. *Level IA: Lower intuitive* with structure $\{a_1<a_2; b_1>b_2\}$. The children usually compare the first terms (a_1 and a_2) without considering the second terms.
2. *Level IB: Medium intuitive* with structure $\{a_1=a_2; b_1>b_2\}$; when the first terms are identical, children compare the second terms to solve the problem.
3. *Level IIA: Lower concrete operational* with composition $\{a_1=b_1; a_2=b_2\}$, with both ratios equivalent to the unit. Children should consider the four terms and use multiplication or division (multiplicative operation).
4. *Level IIB: Higher concrete* with structure $\{a_1/b_1=a_2/b_2\}$. The difference between IIA and IIB is that both ratios are equivalent to a constant different from the unit.
5. *Level IIIA: Lower formal operational* with the form $\{mb_1=b_2, ma_1<a_2\}$. In this case, the terms of a ratio are multiple but not in the other. The student finds the relationship in one ratio and compares it to the other through an additive operation.
6. *Level IIIB: Higher concrete operational,* where children compare any fractions. There is no multiplicative relationship between terms; therefore, they need compare equivalent ratios with the same denominator.

The acquisition of proportional reasoning does not finish until the transition from concrete to formal operations (Butto et al., 2019; Lamon, 2007). Many students do not solve tasks relevant to their logical development until several years later (Van Dooren et al., 2018), even with instruction (González-Forte, 2022). Other research explores proportional reasoning in young children. Thus, Boyer and Levine (2015) used mixture problems like those in the present study, with 8- and 10-year-old children, who found it very difficult to deal with discrete units (glasses of juice and water), while the mixture is a continuum.

Batanero and Hernández-Solís (2023) compared the proportional reasoning level of Costa Rican and Spanish 11-16-year-old students. Few reached the maximum level IIIB, not even in the 10th grade, and their proportional reasoning level was lower than expected in Noelting's (1980a, 1980b) studies. This finding led us to compare this proportional reasoning level with different probability tasks, such as comparing probabilities in spinners described in this paper.

Comparison of Probabilities

Piaget and Inhelder's (1951) research on probability comparison inspired a series of papers summarized in Hernández-Solís et al. (2021) and Jones et al. (2007). Although this research is extensive, most involved comparing probabilities in urns. Some examples are Davies (1965), Goldberg (1966), Hoemann and Ross (197), Pérez- Echeverría et al. (1986), Supply et al. (2020) or Yost et al. (1962).

Some authors used spinners. Falk et al. (1980) asked 61 Israeli children between four and 11 years of age to compare probabilities. They varied the number of favorable and possible cases, using urns and spinners divided into blue and yellow sectors of equal amplitude. They considered the following tasks:

(a) the number of favorable cases was smaller, higher or identical in the higher probability group and

(b) the number of favorable cases was smaller, higher or identical in the lower probability set; both groups are equiprobable, and the number of favorable cases was smaller, higher or identical in one group.

A systematic error was choosing the set with more favorable cases.

Gurbuz et al. (2014) analyzed a teaching intervention with 74 grade 6-grade 8 students. One activity consisted of the comparison of probabilities in two spinners. Some students solved it correctly, comparing areas or ratios. However, other students showed reasoning biases; for example, indicating the speed of the spinner was unknown or reasoning according to the outcome approach.

Research with older students is sparser. Green (1982) assessed probabilistic reasoning in English students aged 11 to 16 with a questionnaire based on Piaget and Inhelder's (1951) experiments. In one item, children should compare two spinners divided into equal amplitude sectors and in another, two spinners divided into sectors with different amplitude.

In both cases, some children compared the number of favorable sectors in the spinner, not considering the area of the sectors. In another item, children did not view equiprobable two equally likely spinners that presented a different distribution of the favorable sectors (interleaved in one spinner and attached in the other*) (distribution bias).* He found the following types of strategies:

(a) comparing the areas of sectors that corresponded to favorable color,

(b) comparing the number of sectors,

(c) comparing ratios between the number of favorable and unfavorable sectors,

(d) position or speed of the needle, and

(e) idea of continuity or separation of favorable sectors.

Cañizares (1997) obtained similar conclusions with 134 students between 10 and 14 years of age.

Maury (1984) investigated the strategies used by 15-16-year-old students in the comparison of probabilities in urns and spinners and three types of tasks:

Table 1. Sample composition by grade & questionnaire

Grade	Age (years)	Questionnaire A	Questionnaire B	Total
6	11-12	35	33	68
7	12-13	26	26	52
8	13-14	31	33	64
8	14-15	26	26	52
10	15-16	27	29	56
Total		145	147	292

1. **Comparison of a single variable:** The number of favorable or unfavorable cases was the same.
2. **Proportionality:** The ratio between the number of favorable cases and the number of possible cases was the same in both bags.
3. **Comparison of two variables:** Non-proportional situations with different numbers of favorable and unfavorable cases.

Relevant arguments for spinners were comparison of areas or favorable/unfavorable cases and ratios. Non-relevant arguments were the distribution of colored sectors in the roulette wheel (distribution bias), which appeared frequently. None of this research reported in detail the achieved level of proportional and probabilistic reasoning at different ages and they only considered the comparison of probabilities in spinners for isolated levels of proportional reasoning.

Relating Proportional Reasoning & Comparison of Probabilities

Pérez-Echeverría et al. (1986) studied the relationship between students' proportional reasoning and their capacity to compare probabilities. The authors gave 10 ratio comparison tasks like those used in our research and 10 problems of comparing probabilities in urns of different reasoning level (Noelting, 1980a) to 20 students aged 12 and 20 aged 17-18.

They found only 12.0% of proportional strategies in the comparison of ratios and 10.0% in the probability tasks. They reported that the students who used a proportional strategy in the easiest items changed to additive strategies in the upper level items, although the use of correct strategies was higher in the older students.

Berrocal (1990) continued the above work and presented a probability task and a proportionality task to 103 students from grade 7 (53 students) and university (50 students). She only reported the correlation between the results of the three tasks. In a second experiment with 305 children aged 11 to 15 years and the same tasks, she examined the task difficulty, informing that the probability task was the most difficult (Berrocal, 1990).

In a previous work (Hernández-Solís et al., 2021), we analyzed the way in which 55 grade 6 primary school Costa Rican children compared probabilities in spinners. However we only used three items and did not analyze the relation of children's performance with their proportional reasoning level. The disposition of favorable sectors in one of the spinners provoked biased responses of the students. In this paper, we analyze probabilistic reasoning in a wider age range and compare it with the students' proportional reasoning level. Additionally, we investigate the bias of considering the distribution of favorable sectors in the spinners as a factor affecting the probability reasoning of participants.

In the present paper we complement all this research and consider the comparison of probability in spinner, which has not been analyzed in relation to the proportional reasoning level.

MATERIALS & METHODS

Sample

There were 292 students in the sample from the last grade of primary education (6th grade of general basic education GBE 11-12-year-olds) and diversified cycle DC grade 10) in Costa Rica. **Table 1** displays the sample composition and the number of students who answered each questionnaire.

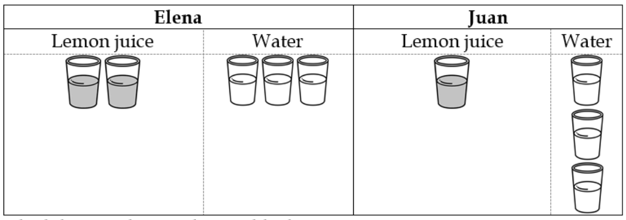
All the students came from a private school in the City of Cartago; in total, 13 groups of students composed the sample. This school was selected because of its prestige and tradition with 65 years of service and because there were groups of students from primary and secondary education. Students in the school comes from medium and high social and economic background.

The students in the sample studied probability and proportionality since the beginning of primary education according to the guidelines in effect (MEP, 2012). In grade 6, students remembered the intuitive ideas previously acquired and studied the Laplace rule. In grade 8, they revised all the probability ideas introduced in previous grades. In grade 9, they studied the frequentist definition of probability and the law of large numbers and in grade 10, the properties of union and complement. The study of rational numbers started in primary education, including the concept of fractions, their representation, order and operation, and equivalent fractions. In grade 7, children used natural and integer and a bit of inverse proportionality. They worked the rational number and their properties in grade 8, and in the following years, they applied proportional reasoning in problem-solving.

Table 2. Type of item, reasoning levels (Noelting 1980a, 1980b), & questionnaire (Y: Years & M: Months)

Item	Item type	Composition (a_1, b_1) vs. (a_2, b_2)	Proportional reasoning level	Age (Y, M)	Questionnaire
1	Ratios	(2, 3) vs. (1, 3)	IA	(3, 6)	A
2	Ratios	(5, 1) vs. (5, 4)	IB	(6, 4)	B
3	Ratios	(2, 2) vs. (4, 4)	IIA	(8, 1)	A
4	Ratios	(3, 1) vs. (6, 2)	IIB	(10, 5)	B
5	Ratios	(3, 1) vs. (4, 2)	IIIA	(12, 2)	A
6	Ratios	(3, 2) vs. (4, 3)	IIIB	(15, 1)	B
7	Spinners unbiased	(3, 2) vs. (5, 2)	IA	(3, 6)	B
8	Spinners unbiased	(4, 1) vs. (4, 3)	IB	(6, 4)	A
9	Spinners unbiased	(2, 2) vs. (3, 3)	IIA	(8, 1)	B
10	Spinners unbiased	(2, 6) vs. (1, 3)	IIB	(10, 5)	A
11	Spinners unbiased	(3, 6) vs. (1, 3)	IIIA	(12, 2)	B
12	Spinners unbiased	(3, 4) vs. (4, 5)	IIIB	(15, 1)	A
13	Spinners biased	(3, 3) vs. (4, 4)	IIA	(8, 1)	B
14	Spinners biased	(4, 8) vs. (2, 4)	IIB	(10, 5)	A
15	Spinners biased	(8, 4) vs. (6, 2)	IIIA	(12, 2)	B
16	Spinners biased	(5, 4) vs. (4, 3)	IIIB	(15, 1)	A

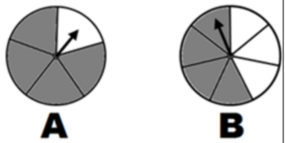
Item 1. Elena & Juan made some lemonade. Elena mixed two glasses of lemon juice with three glasses of water. Juan combined one glass of lemon juice with three glasses of water. All glasses contained the same amount of liquid. Look at the picture.



Which lemonade tasted more like lemon?
( ) Elena's.
( ) Juan's.
( ) Both were identical.
( ) I do not know.
Explain why you gave that answer.

Figure 1. Item 1 (questionnaire A) (Source: Authors' own elaboration)

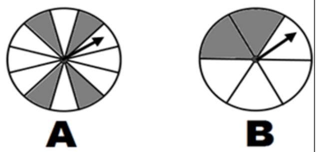
Item 8. The spinner A is divided into five equal parts (four are colored black and one is colored white) and the spinner B is divided into seven equal parts (four are colored black and three are colored white). Look at drawing.



If you spin the arrow, which of the two spinners gives the highest probability that the arrow lands on black? Point out the correct answer.
( ) Spinner A.
( ) Spinner B.
( ) Both spinners have the same probability.
( ) I do not know.
Explain why you give this answer:

Figure 2. Item 8 (questionnaire A) (Source: Authors' own elaboration)

Item 14. The spinner A is divided into 12 parts of equal area (four are colored black and eight are colored white) and the spinner B is divided into six parts of equal area (two are colored black and four are colored white). Look at the drawing.



If you spin the arrow, which of the two roulette wheels gives the highest probability that the arrow lands on the black color? Point out the correct answer.
( ) Wheel A.
( ) Wheel B.
( ) Both roulette wheels have the same probability.
( ) I do not know.
Explain why you give this answer:

Figure 3. Item 14 (questionnaire A) (Source: Authors' own elaboration)

Questionnaires

In **Table 2**, we present the characteristics of each item. These students solved one of two questionnaires, A and B, each containing three ratio comparison problems similar to those used by Karplus et al. (1983), Noelting (1980a, 1980b), and Tourniaire and Pulos (1985) (**Figure 1**).

Also, each questionnaire included three probability comparisons in spinners problems with adjacent colored sectors and two probability comparisons in spinners problems with alternated colored sectors (**Figure 2**).

In each questionnaire, we considered three increasing proportional reasoning levels; together, six reasoning levels appear in both questionnaires (**Figure 3**).

The construction of the questionnaires was rigorous, following AERA et al. (2014) recommendations. We assured content validity by expert judgment who collaborated to select the items (between three different versions for each item). The final items included in the questionnaire were those with higher median and mean scores (four points or more) and less variability in the experts' scoring on a 5-point Likert scale. Cronbach's alpha reliability coefficient was .744 for questionnaire A and .765 for questionnaire B. Each questionnaire was given to half the children in each school grade to ensure that an approximately equal number of students per grade answered each item.

We reproduce three items in **Figure 1** to **Figure 3** as examples, where two ratios or probabilities (a_1, b_1) and (a_2, b_2) should be compared. In items 1 to 6, the first term of each pair is the antecedent or dividend of the ratio (glasses of juice/glasses of water), and the second term is the consequent or divisor in the comparison of ratios items.

In the remaining items, the terms represent the number of favorable and unfavorable cases. In items 13 to 16 we additionally introduced a distractor, as the favorable sectors are attached in one spinner and interspersed in the other. The aim was to check if the students manifested the distribution bias described by Cañizares (1997), Green (1983), and Maury (1984).

Strategies Categories

We studied the students' written responses to the questionnaires with content analyses (Krippendorff, 2018), a method that helped refine the initial analyses categories taken from previous research (Cañizares; 1997; Cañizares et al., 1997; Green, 1983; Maury, 1984). We classified the students' strategies to solve the tasks as correct or incorrect. The procedure was correct when it involved a mathematical acceptable method to solve a given problem. With successive revisions and discussion we refined the classification. An author coded the responses, and another recoded the responses of 20 students to compute the inter-coder reliability. We obtained values of Cohen's kappa=.9739 for the coding of responses and .9186 for the coding of strategies. We now describe the strategies and include responses of students as example. We denote the students by Sx, where x is the student's order in the data file.

Comparing totals in each ratio (analyzing the number of possible events in probability tasks). Noelting (1980b) did not report this strategy, which is incorrect for all the items:

> S205: She has fewer glasses in (item 4, option A).

> S136: In spinner A is harder to win, since there are more divisions than in B (item 10, option B).

Comparing the first terms "a" of ratios (analyzing the favorable cases). This procedure provides correct results only when the second terms of the ratios (unfavorable sectors) are identical (b_1=b_2). This strategy, typical of Noelting's (1980a, 1980b) lower intuitive level (IA), provides correct answers to 1 and 7 (S2, S291). Other students incorrectly used this method, for example, S50, who did not notice that spinner A also contained more white parts. S8 provided a similar response in the comparison of ratios.

> S2: Because the more lemon juice you put in the water, the more it tastes like lemon juice (item 1, option A).

> S291: There are more black pieces (item 7, option B).

> S8: Because there are more glasses with lemon (item 3, option B).

> S50: Because there are more black parts (item 10, option A).

Comparing the second terms "b" of ratios (studying the unfavorable events). This strategy is valid when first terms identical (a_1=a_2) because "b" is the reciprocal of "a". The procedure is typical of Noelting's (1980a, 1980b) intermediate intuitive level (IB) and gives correct answers to items 2 and 8 (see S274 and S57). However, S154 assumes equiprobability because he did not compare the unfavorable cases in item 7, and S12 did not consider the number of lemon glasses.

> S274: Because he used less water (item 2, option A).

> S57: Both spinners include four black spaces, but in B, there are more white spaces and then A is more likely (item 8, option A).

> S12: There will be less water (item 3, option A).

> S154: Because both have the same amount (2 parts) (item 7, option C).

Comparing the differences between the terms of each ratio (Comparing the differences between favorable and unfavorable events). Noelting (1980a, 1980b) remarked that this strategy implies that the student perceives the ratio as a whole. It is valid in items of lower levels; for example, in items 1 and 2, 7 and 8 of the questionnaire (see S163 and S3), although it is incorrect in terms 6 12, which requires a multiplicative strategy (S178, S123).

> S163: Juan has more lemonade, but Elena's lemonade is more concentrated because she did not dilute it as much (item 2, option A).

> S3: There are fewer whites than blacks in A (item 8, option A).

> S178: In both mixtures there is one more glass of lemon; the difference is that Juan would have more liquid (item 6, option C).

> S123: Because in both mixtures there is one more white than black (item 12, option C).

Ratio of equivalence to the unit. The student compares one ratio (a_1/b_1) with the other (a_2/b_2), discovering both are equivalent to the unit (equiprobability in comparison of probabilities). While the previous strategies involve combining the terms in the same ratio in this case both ratios should be considered. It corresponds to Noelting's (1980a, 1980b) lower concrete operational level (IIA) and gives appropriate answers to items 3 and 9 (S107; S275). However, is incorrectly used in other items (S279, S106).

S107: Both use exact amounts of juice and water (item 3, option C).

S275: Because they have the same amount of painted and non-painted zones (item 9, option C).

S279: Because they are using the same amount of lemon and water (item 2, option C).

S106: Same number of winners and losers (item 2, option A).

Equivalence between ratios. The student compares one ratio with the other with a multiplicative operation, finding that they are equivalent. This strategy leads to correct answers only when the ratios belong to the same equivalence class of fractions. The procedure corresponds to Noelting's(1980a, 1980b) higher concrete operational level (IIB) and solves items 4 and 10 (S217, S71). It was also applied incorrectly (e.g., S212, S35).

S217: Because Elena's amount of lemonade is smaller, and so is the amount of water, Juan's are as if multiplied by 2 (item 4, correct option).

S71: Although A has double white spaces, it also has double black spaces (item 10, option C).

S212: Because both mixtures have the same proportion (item 12, option C).

S35: There is the same number of white sectors for each black (item 5, option C).

Correspondence between the ratio terms. The students construct a proportionality criterion between the terms of the first ration (a_1/b_1) to determine whether the relationship in the other ratio (a_2/b_2) is smaller or larger. This strategy gives correct answers provided that two of the four terms to compare are multiples and matches to the lower formal operational level (IIIA) and solves items 5 and 11. Then, S224 build a correspondence between the terms of each ration than later compares. It was always used correctly.

S80: There are three juices for one water, while in Juan's case, there are two juices for each glass of water (item 5, option A).

S224: The simplified fraction of its parts is smaller (3/6=1/2>1/3) (item 11, option A).

Proportionality. Ratios are reduced to common denominator fractions and compared. With this strategy, we can compare any ratio, and gives correct answers for any comparison task. It corresponds to Noelting's (1980a, 1980b) higher formal operational level (IIIB) and solves all the questionnaire items. It was always correctly used.

S255:8/12 and 6/8; then 2/3>3/4 (item 15, option A).

S205: Because Elena's ratio is 21/35 and Juan's is 20/35. That is to say, Elena's concentration is higher (item 6, option A).

In addition, we found the following incorrect strategies in the comparison of probability in spinners:

Equiprobability bias. When all the events in a random experiment are considered equiprobable (Lecoutre, 1992). For example, S46 and S62 assume the equiprobability of white and black, no matter the number of sectors of each color.

S46: I think the probability is the same as both spin (item 12, option C).

S62: It could stop either in white or black in the same way (item 8, option C).

Outcome approach. Some students do not interpret the problem in a probabilistic way but deterministically and understand they should predict the results of the random experiment. Konold (1989) described this reasoning as the outcome approach. For example, S174 assumes equiprobability (option C) in thinking the result depends on chance, and then any outcome is equiprobable, no matter the spinners' composition.

S174: Both, since the spinners depend on chance, and we cannot predict, which will stop (item 7, option C).

Physical considerations. The student associates the conditions of the experiment with the probability of a given event. Some factors are the speed at which the roulette wheel spins or the force with which it is spun (S112). Moreover, in items 7 to 12, the arrows are placed in different positions, and this fact influenced some students' answers (S208).

S112: It depends, as it is a spinner, and we can pull the arrow at different speeds. If A is pulled slowly, it may stop anywhere, the same in B. It depends on the force and speed of the arrow (item 8, option C).

S208: Since it lacks two jumps to reach the black. On A, it's three more to go! (item 12, option B).

Distribution of colored sectors. In items 13 to 16, black and white sectors alternate in one spinner, being adjacent in another, to assess whether this fact influenced the students' answers. We found students who assigned a higher probability to the spinner, where the sectors of the same color were attached to each other (S223); and others gave a higher probability when the sectors alternated. For example, although S224 calculated 3/3=4/4, recognizing the equivalence to the

Table 3. Percentage of students selecting correct answer in comparison of ratios by school grade

Item	Noelting level	Grade				
		6	7	8	9	10
1	IA	77.2	80.7	90.4	76.8	88.9
2	IB	87.9	92.3	78.8	88.4	79.3
3	IIA	68.6	69.2	64.5	69.2	70.4
4	IIB	27.2	38.4	39.4	42.3	51.7
5	IIIA	11.4	15.3	19.4	30.7	37.0
6	IIIB			3.0	7.7	10.3

Table 4. Percentage of students selecting correct answer in comparison of probabilities by school grade

Item	Noelting level	Biased item	Grade				
			6	7	8	9	10
7	IA		63.6	76.9	75.8	80.8	82.8
8	IB		82.9	100.0	80.6	80.8	88.9
9	IIA		66.7	65.4	81.8	88.5	72.4
10	IIB		34.3	61.5	48.4	69.2	70.4
11	IIIA		60.6	53.8	57.6	65.4	58.6
12	IIIB		31.4	30.8	19.4	34.6	29.6
13	IIA	x	51.5	50.0	54.5	46.2	48.3
14	IIB	x	31.4	26.9	41.9	53.8	48.1
15	IIIA	x	36.4	53.8	39.4	50.0	41.4
16	IIIB	x	37.1	53.8	35.5	34.6	40.7

unit in item 10, he assigned a higher probability to roulette B because the colors were "interspersed".

> S223: Because all the black triangles are close together, there is a greater probability that the arrow will fall there (item 13, option A).

> S224: Because it is divided into more sections and also interspersed, roulette B is more likely to land on black (item 10, option B).

RESULTS

In this section we reply to the research questions posed in the introduction.

Correct Responses in the Comparison of Probabilities

First, we analyze the percentage of students in each grade that correctly solved each item by grade. In **Table 3** we present the results in the comparison of ratios and in **Table 4** the results in the comparison of probabilities.

There is a general improvement with grades in levels IA to IIA items, although not consistently. The percentage of correct response is in general higher in probability tasks because many students compared the areas colored white and black on the spinners instead of ratios of favorable and unfavorable cases. This result contradicts the findings of Batanero et al. (in press) and Berrocal (1989) who used comparison of probabilities in urns. We then assume that the students understand better the idea of ratio as comparison of part to whole (spinner) than as comparison of part to part (urns) as suggested by Cañizares (1997).

The percentage of correct responses decreases when the proportional reasoning level increases in the items in all the grades and more in the comparison of ratios. We remark the different percentage of correct responses in level IIIA and IIIB in both types of tasks. However, when comparing the items corresponding to the same level of probabilistic reasoning with and without bias, the number of correct answers in the latter decreased, because some students were influenced by their mistaken beliefs. This tendency does not continue in items IIIB and IIIB biased. On the one hand, the students who used proportionality in these items and reached the higher proportional reasoning level were not confused by erroneous beliefs. On the other hand, many students compared areas instead of working with fractions in these items and could solve the problem with a lower reasoning level.

Strategies in the Comparison of Probabilities

We secondly investigate the correct strategies in the comparison of probabilities tasks (see **Table 5**).

There is a predominance of comparison of areas in all the items and grades. However, when comparing the biased and non-biased items of the same reasoning level, there is less proportion of comparison of areas in the biased items. Probably, when the same color sectors are interleaved, the total amount of area of the same color is not so clearly perceptible. We obtain a similar conclusion in the comparison of unfavorable events, only used in items IB with a smaller percentage than the comparison of areas.

The comparison of favorable cases was only applied in level IA and with less frequency than the comparison

Table 5. Percentage of correct strategies in comparing probabilities by item & grade

Strategy	Item	Grade 6	7	8	9	10
Comparing favorable cases	7 (IA)	30.3	19.2	27.3	11.5	24.1
Comparing unfavorable cases	8 (IB)	20.0	23.1	22.6	15.4	11.1
Comparing differences	7 (IA)	3.0	23.1	15.2	11.5	6.9
	8 (IB)	37.1	11.5	12.9	3.8	18.5
Equivalence to unit	9 (IIA)	27.3	19.2	36.4	30.8	24.1
	13 (IIA)	24.2	15.4	24.2	15.4	13.8
Equivalence to ratio	10 (IIB)	5.7	2.9	12.9	19.2	3.7
	14 (IIB)	8.6		6.5	11.5	3.7
Correspondence	8 (IB)				3.8	
	10 (IIB)	2.9				
	11 (IIIA)				3.8	3.4
	14 (IIB)	2.9				
Proportionality	7 (IA)			6.1	7.7	6.9
	8 (IB)	2.9	11.5	3.2	3.8	3.7
	9 (IIA)	6.1	7.7	12.1	15.4	13.8
	10 (IIB)	8.6	5.7	16.1	15.4	22.2
	11 (IIIA)	6.1	11.5	21.2	19.2	20.7
	12 (IIIB)	5.7	2.9	3.2	15.4	11.1
	13 (IIA)	6.1	15.4	9.1	15.4	13.8
	14 (IIB)	5.7	2.9	19.4	11.5	7.4
	15 (IIIA)	3.0	7.7	9.1	7.7	13.8
	16 (IIIB)	5.7	2.9	12.9	7.7	
Comparing areas	7 (IA)	36.4	50.0	30.3	38.5	31.0
	8 (IB)	17.1	50.0	22.6	38.5	37.0
	9 (IIA)	45.5	57.7	27.3	23.1	48.3
	10 (IIB)	37.1	34.3	25.8	30.8	66.7
	11 (IIIA)	30.3	46.2	33.3	53.8	31.0
	12 (IIIB)	34.3	37.1	32.3	38.5	37.0
	13 (IIA)	15.2	23.1	24.2	19.2	24.1
	14 (IIB)	22.9	22.9	12.9	34.6	40.7
	15 (IIIA)	21.2	38.5	30.3	23.1	20.7
	16 (IIIB)	25.7	31.4	22.6	11.5	33.3

of areas. Similar behavior was noticed in the comparison of unfavorable cases, only used in level IB. This fact suggests that the students understand better the idea of ratio as comparison of part to whole than as comparison of part to part. The analyses of differences was only used in items IA and IB.

The correspondence strategies were very scarce in all the items and grades, while students used proportionality in a small percentage, that increased with the grade and item level. This strategy was also less frequent in the biased items.

All the correct strategies found in our previous study (Batanero et al., in press) appear in this case. However, the frequency of comparison of favorable or unfavorable cases or equivalence to unit is lower because students tend to use the comparison of areas, that cannot be applied in the case of urns.

Table 6. Mean percentage of incorrect strategies by item in comparison of probabilities

Strategy	Items	Grade 6	7	8	9	10
Comparing totals	7-16	8.7	5.7	6.0	3.8	4.9
Comparing favorable cases	8-16	10.6	7.9	9.5	6.4	6.0
Comparing unfavorable cases	7 & 9-16	3.6	3.5	4.6	3.0	1.9
Comparing differences	9-16	10.0	4.6	8.6	2.7	4.7
Equivalence to unit	7, 8, 10-12, & 14-16	0.4		0.8		0.5
Equivalence to ratio	7, 8, 11, 12, 15, & 16	0.0	1.0	0.8	1.0	0.9
Equiprobability bias	7-16			1.3	1.5	0.3
Outcome approach	7-16			1.2	0.4	0.4
Physical considerations	7-16	4.1	1.2	2.2	3.1	2.9
Distribution bias	13-16	23.6	17.7	20.2	23.1	30.4
Confuse arguments	7-16	3.3	2.4	2.2	1.5	1.4
No argument	7-16	4.4	1.7	5.6	12.3	6.7

Incorrect Strategies

In **Table 6** we present the mean percentage of incorrect strategies per item. We computed these mean percentages by dividing the sum of percentages of students using each strategy in different items by the number of items in which the strategy appeared. For example, 10.6% in comparing favorable cases in grade 6 means that 10.6% of students on average (mean) incorrectly used this strategy in items 8 to 16.

The frequency of incorrect strategies is very similar to that found in the comparison of urns in our previous study (Batanero et al., in press). The exception is the distribution bias, which has a remarkable frequency in the biased items and do not apply in the comparison of urns.

Comparison of totals appears on all items and is always a wrong strategy. Its frequency was low and mainly happened in tasks IIB and IIIB (around 10.0%). Incorrect comparison of favorable cases (which is only correct in item 7) was also frequent, especially for students in grade 6 (30.0%) in items IIIA (both biased and unbiased items)

These same students incorrectly compared the differences between favorable and possible cases more frequently than in the other grades in level IIB and higher items.

It is worth noting the responses based on the physical layout of the colored sectors, used in the biased items in a high proportion in all grades. It stands out in grade 10, with more than 27.0% in every one of these items. Although the equiprobability, outcome approach and physical considerations biases are present in all items, their frequency is negligible.

Table 7. Percentage of students according to proportional reasoning level achieved

Level	Grade				
	6	7	8	9	10
0	11.8	11.5	14.1	13.5	10.7
IA	11.8	11.5	12.5	5.8	10.7
IB	30.9	26.9	21.9	23.1	17.9
IIA	29.4	25.0	21.9	23.1	19.6
IIB	13.2	17.3	18.8	17.3	21.4
IIIA	2.9	5.8	9.4	13.5	14.3
IIIB		1.9	1.6	3.8	5.4

Table 8. Percentage of students according to proportional level achieved in comparison of probabilities by grade

Level	Grade				
	6	7	8	9	10
0	19.1	7.7	18.8	15.4	10.7
IA	5.9	9.6	3.1	5.8	7.1
IB	26.5	21.2	14.1	5.8	10.7
IIA	19.1	17.3	15.6	17.3	17.9
IIB	7.4	13.5	18.8	26.9	25.0
IIIA	13.2	17.3	23.4	23.1	19.6
IIIB	8.8	13.5	6.3	5.8	8.9

Reasoning Levels

Finally, to analyze which probabilistic and proportionality reasoning level do students reach in each grade, in **Table 7** and **Table 8** we present the percentages of students according to the level of proportional and probabilistic reasoning achieved by grade and country. We allocate a student in a particular level when he/she has correctly solved the item associated with the level (correct answer and argument) and all the items of lower levels. Level 0 means that the student did not solve any of them correctly either because they failed in the strategy or the answer. We observe a significant percentage of these students in all grades, which indicates the difficulties these students still have with solving the tasks. The percentage is higher in the comparison of probabilities in spinners in most grades.

The reasoning level increases as expected with the school grade. Thus, while the majority of students were located in levels IB and IIA in grade 6, in grade 10 the majority were located in levels IIA to IIIA. There were few students reasoning at the upper level IIIB, although the average age hypothesized by Noelting (1980b) to reach this level is 15 years and 1 month and the students in grade 10 are 15-16-year-olds.

To better analyze the association between the probabilistic and proportional reasoning with the school grade, in **Table 9** we display the Pearson's correlation coefficient between these variables.

All these correlations are statistically significant but of small intensity except the correlation between grade and proportional level. Moreover, there is a stronger association between the proportional and probabilistic

Table 9. Pearson's correlation coefficient between probabilistic & proportional reasoning & grade

		PLL	PCL
Grade	Pearson's correlation	.097	.124*
	p-value	.196	.034
Proportional level	Pearson's correlation		.209*
	p-value		.000

Note. PLL: Proportional level & PCL: Probabilistic level

Table 10. Pearson's correlation coefficient between number of correct responses in problems of mixtures & comparison of probabilities (biased & non-biased items) by grade

		Number of correct responses		
		Mixtures	Spinners	Spinners biased
Grade	PC	.137*	.126*	.069
	p-value	.019	.031	.240
Mixtures	PC		.207**	.135*
	p-value		.000	.021
Spinners	PC			.249**
	p-value			.000

Note. PC: Pearson's correlation

reasoning than the corresponding to any of these variables and grades.

Consequently, it is important to reinforce the students proportional reasoning to help them succeed in comparing probabilities. Since correlation is a symmetrical property, conversely, improving the students' probabilistic reasoning level will help them develop their proportional reasoning.

In **Table 10** we display Pearsons' correlation coefficients between the number of correct responses (correct option and correct arguments) in the three types of items (comparing mixtures and comparing probabilities in biased and non-biased spinners).

We observe that all the correlations are positive and statistically significant, although of small intensity. The highest correlation appears between the comparison of spinners in biased and non-biased items. This means that as the students solve more non-biased items in the comparison of probability he or she also solve more biased items. There is also an important correlation between the number of mixture problems solved and the number of probability comparison in non-biased items. Therefore, solving proportional problems help solve the probabilistic problems in non-biased items. The association is much lower with the biased items, so that proportional reasoning only does not help solving biases related to the disposition of sectors in the spinners.

DISCUSSION & CONCLUSIONS

In this study, we jointly evaluated the performance in comparing probabilities in spinners and the proportional reasoning level in a sample of grade 6 to grade 10 Costa Rican students. We analyzed the open-ended responses of 292 students to three types of items:

comparing mixtures and comparing probabilities in biased and non-biased spinners. We analyzed the percentages of correct selection to the items, strategies in the comparison of probability and proportional and probabilistic reasoning levels.

As regards the selection of the correct response, items of levels IA, IB, and IIA were extremely easy for the students, since we obtained high percentages of correct responses in these items. The explanation is that they can be solved by comparison only of one variable or identifying the unit ratio, which is clearly visible in items IIA. The difficulty increased in the following levels, although there was a general improvement with grade in all the tasks.

Our results contradict the findings of Batamero al. (in press) and Berrocal (1989) who used comparison of probabilities in urns since in these studies the comparison of probabilities was harder than the comparison of ratios. We then assume that the students understand better the idea of ratio as comparison of part to whole (spinner) than as comparison of part to part (urns) as suggested by Cañizares (1997).

We additionally found lower percentages of correct responses in the biased and non-biased items for the same reasoning level. This fact points to the existence of the distribution bias reported by Cañizares (1997), Green (1983), and Maury (1984).

As regards the strategies in the comparison of probabilities, we did not observe the bias reported by Falk et al. (1980) consisting of in systematically choosing the set with more favorable cases. Although this strategy was incorrectly used in many items the percentage of use only was higher than 10.0% in grade 6 and item 11. Moreover, in general, the selection of the option depended on the characteristics of the item and thus, in level IB items students opted for the spinner with more unfavorable sectors. In items of level IIA they decided equiprobability, and they adapted their strategies in the remaining items. These strategies were also reported by Cañizares (1997), Green (1983), and Maury (1984).

We also coincided with Gurbuz et al. (2014) that many students correctly solved the tasks by comparing the areas of black and white sectors a strategy, which is specific for the comparison of spinners and do not appear in the comparison of probabilities in urns.

We also noticed the influence of some biases. In coincidence with Cañizares (1997), Green (1983), Gurbuz et al. (2014), and Maury (1984). The equiprobability, outcome approach and influence of physical considerations a9ppeared in our sample in very few students. However, there was a much strong influence of the distribution of colored sectors in the spinners in the items 13 to 16.

Finally, the results suggest that the level of proportional reasoning in the comparison of probabilities increase with the grade. However, the age at which the higher levels are reached later than assumed by Noelting (1980b). We also observed correlation between proportional and probabilistic reasoning, which was higher than that between any of these variables and school grade. The number of correct responses and arguments to the three types of items used in the study are also correlated. However, the correlation of the number of problems with grade and the number of biased items solved with the number of correct proportional problems is very small.

Since our sample is intentional and restricted to only a country, there is a need to replicate this research with other sample of students to evaluate the extent of the tendencies reported in this study.

Anyway, the results point to the need to reinforce the proportional reasoning of students when approaching the teaching of probability and that conversely increasing the probabilistic reasoning help develop proportional reasoning. Moreover than probability problems can be used as examples of proportional problems and example-based practice is beneficial for students with less prior knowledge of proportions (Begolli et al., 2021).

Teachers should consider these results in organizing their teaching of probability and proportionality in secondary school levels. This also involves the need of improving the teachers' understanding of ratio and proportion and to teach proportionality (Burgos & Godino, 2021, 2022).

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