MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(6), em2450
ISSN:1305-8223 (online)
OPEN ACCESS
Review Article
https://doi.org/10.29333/ejmste/14588

Advances and trends in research on mathematical working spaces: A systematic review

Darlis Panqueban [1], Carolina Henríquez-Rivas [1*], Alain Kuzniak [2]

[1] Department of Mathematics, Physics and Statistics, Faculty of Basic Sciences, Universidad Católica del Maule, Talca, CHILE
[2] UFR of Mathematics, André Revuz Didactic Laboratory, Université Paris Cité, Paris, FRANCE

Received 27 December 2023 ▪ Accepted 23 April 2024

Abstract
This study addresses the research developed around the theory of mathematical working spaces (MWS). To this end, factors such as researchers' country of origin are considered, as well as methodological characteristics, proposed objectives in each study, specific theoretical factors, and the links between MWS and other theoretical perspectives. This systematic review follows the guidelines stated by reporting items for systematic reviews and meta-analyses for documenting each stage of the review. In total, 102 studies were considered from databases including Dialnet, SciElo, Scopus, and Web of Science. The results help to illustrate research trends in MWS theory, as well as the possibility of fertile ideas for new studies and future lines of research for those interested in this theoretical approach.

Keywords: mathematical working spaces, theory, mathematics education, tasks, systematic review

INTRODUCTION

In recent decades, the field of mathematics education research has undergone the development of diverse theoretical and methodological positions, lines of research, and research communities that contribute to addressing and explaining issues from different points of view (Bikner-Ahsbahs et al., 2015; Laborde, 2007; Lerman, 2020). These contributions reflect the evolution and thematic vitality of the discipline, especially its development in recent years (Gaona & Arévalo-Meneses, 2023). From this context, a growing interest has emerged in carrying out analyses of literature centered on the state of the art of research focused on specific mathematical domains or theoretical approaches (e.g., Cevikbas et al., 2022; Schoenherr & Schukajlow, 2023; Vásquez et al., 2023). The present study considers research developed around a theory known as mathematical working spaces (MWS), whose results are presented in this article. The aim of this analysis is to contribute to a better understanding of the current research landscape in MWS and to inspire new studies.

The theory of MWS is currently in a period of advancement, which leads to the consideration of both theoretical and methodological aspects of research that is underpinned by this framework (Henríquez-Rivas et al., 2021; Kuzniak y Nechache, 2021; Nechache & Gómez-Chacón, 2022). Its main purpose within mathematics education is to describe, understand, and transform mathematical work in educational contexts (Kuzniak et al., 2022).

The beginnings of this theory trace back to the 1990s, with the first works emerging from researchers in France and centered on geometry teaching among primary school teachers (Houdement & Kuzniak, 1996). To this effect, the authors define a conceptual framework that organizes geometry around what they term geometric paradigms (Houdement & Kuzniak, 1999). With the theoretical expansion of Geometric working space to MWS (Kuzniak, 2011), research spread to other domains (e.g., analysis, probability, and algebra), reaching a notable level of collaboration and development by researchers from diverse countries, mainly in Europe and the Americas (Derouet & Parzysz, 2016; Montoya-Delgadillo & Vivier, 2014, 2016).

A particularly important venue for the development of MWS has been Mathematical Working Spaces Symposium, held in various countries beginning in 2009

✉ darlis.panqueban@alumnos.ucm.cl ✉ chenriquezr@ucm.cl **(*Correspondence)** ✉ alain.kuzniak@univ-paris-diderot.fr

Contribution to the literature

- The results of this review reveal how MWS is utilized for the study of mathematical work of students, future teachers, in-service teachers, and teacher educators in distinct school levels and contexts.
- One contribution of this research is the identification of research trends, productive ideas, and future lines of research from the perspective of MWS.
- Another contribution of this study is related to expanding the possibilities of proposing research that considers theoretical connections between MWS and other perspectives.

(Cyprus 2009, France 2010, Canada 2012, Spain 2014, Greece 2016, Chile 2018, France 2022), which has provided space for promoting the role of research in mathematics learning and teaching (Gómez-Chacón et al., 2015, 2017). The two most recent conferences have been organized around four thematic lines:

(1) mathematical work and MWS,

(2) specificity of tools and signs in mathematical work,

(3) genesis and development of mathematical work: the role of the teacher and interactions, and

(4) role and use of tasks in mathematical work (Derouet et al., 2023; Montoya-Delgadillo et al., 2019).

THEORETICAL FRAMEWORK

Mathematical Working Spaces

In this section, the theoretical framework that underlies the research reviewed in this study is presented, followed by the research questions formulated thereupon.

MWS is a didactic theory that is unique from other approaches currently used in mathematics education (Radford, 2017); it relates mathematical content by intricately combining epistemological aspects of mathematics and cognitive processes of subjects. The richness of this theoretical corpus has facilitated its progress and deeper development, both in theoretical and methodological aspects, which is especially highlighted by the research undertaken in the past several years.

MWS theory initially emerged for geometry teaching (Houdement & Kuzniak, 1999); subsequently, it has been further developed in distinct mathematical domains (Kuzniak, 2011). This theory addresses the description, understanding, and educational formation of mathematical work on the part of educational actors to enable working with these phenomena (Kuzniak, 2022). Thus, the objective of the theory is didactic study of the mathematical work in which students and teachers participate to contribute to the understanding of the work done in solving tasks, as well as allowing for the characterization of the routes taken in their resolution (Kuzniak et al., 2016b).

In MWS theory, tasks occupy an important place and, therefore, have important implications for associated research, as they are understood as the medium for solving problems (Kuzniak, 2022). While tasks are not an explicit component of the model, they are understood as activators of the work of the individual in a specific institution (Kuzniak, 2011). For this reason, the study of tasks, their design, and their implementation is a topic that has gained increasing relevance in MWS research.

MWS theory considers the epistemological principles of the objects that are studied within a mathematical domain (e.g., geometry and probability) (Kuzniak, 2011; Montoya-Delgadillo & Vivier, 2016). Likewise, the theory integrates the human (or social) component, which entails considering a cognitive dimension related to the epistemological dimension (Kuzniak, 2022). These two dimensions, termed epistemological and cognitive planes, intend to capture the mathematical content of the domain being studied and the cognitive activity of the individual when they acquire, develop, or utilize those mathematical contents (Kuzniak, 2011).

The epistemological plane has three interacting components: the representamen, associated with a concrete and tangible group of symbols based on the interpretations and relations constructed by the individual; artifacts, such as drawing tools, software, or a symbolic system, utilized as an instrument for action; and the referential, a theoretical reference system corresponding to definitions, properties, and theorems. The cognitive plane is organized around three processes: visualization, related to the deciphering and interpretation of signs; construction, a process based on the actions triggered by the artifacts utilized and the associated usage techniques; and proof, a process understood as all discursive reasoning that allows the formulation of deductively-organized arguments, definitions, hypotheses, and conjectures, along with enunciating counterexamples, with the support of the referential (Kuzniak, 2022; Kuzniak et al., 2016b). The connection between these planes occurs through semiotic, instrumental, and discursive geneses, which enable coordinating and specifying the nature of mathematical work in diverse educational and institutional contexts (Kuzniak, 2011).

Semiotic genesis represents the relation between the mathematical object and the cognitive process of visualization that gives it meaning. In instrumental

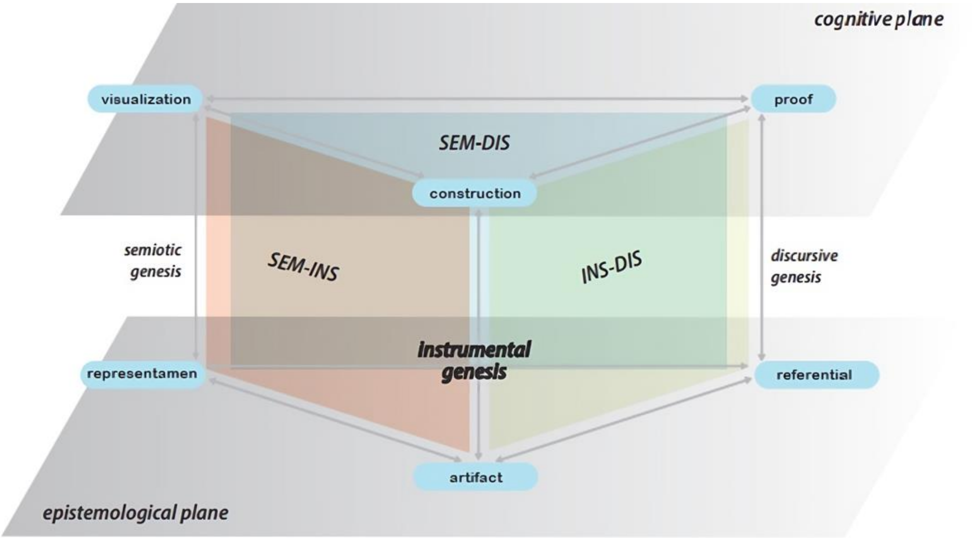

Figure 1. MWS diagram (adapted from Kuzniak et al., 2016b)

genesis, artifacts become operational through the construction carried out by the individual. Discursive genesis relates the referential and the proof process. Thus, in the resolution of a given task, these three geneses and the relations among the components of the planes can interact to give meaning to the mathematical work of the individual.

Coutat and Richard (2011) and Kuzniak and Richard (2014) recognize the concept of vertical planes, understood as the interactions between two geneses and the associated components. In these interactions, three different vertical planes are identified: using artifacts in the construction of results under certain conditions or in the exploration of semiotic representations (Sem-Ins vertical plane); the process of proof or the validation of a construction based on experimentation using an artifact (Ins-Dis vertical plane); and coordinating the process of visualization of represented objects with validation logic (Sem-Dis vertical plane) (Kuzniak et al., 2016a). The relations among the planes, components, geneses, and vertical planes are illustrated in the following diagram (**Figure 1**).

In this manner, MWS research is based on studying and understanding the dynamic of mathematical work through the role of each of these geneses and their interactions (Kuzniak, 2018), which is termed circulation in MWS (Montoya-Delgadillo et al., 2014). Thus, MWS, with its planes, components, and geneses, is a methodological and analytical tool used to identify the different phases of the process of problem-solving and to describe the evolution of mathematical work (Kuzniak & Nechache, 2021; Nechache & Gómez-Chacón, 2022). In the analysis of circulations, changes between two mathematical domains can be identified (Montoya-Delgadillo & Vivier, 2014), as well as barriers, difficulties, and restraints (Henríquez-Rivas & Montoya-Delgadillo, 2016; Kuzniak, 2022) and other cases in which varied interactions exist, which is known as complete mathematical work (Kuzniak et al., 2016a).

Finally, three types of MWS are distinguished in the theory depending on users, their position in a school institution, and their role in the implementation of the school curriculum (Gómez-Chacón et al., 2016a, 2016b, 2016c). These include the referential MWS, related to people or institutions responsible for the school institution in accordance with mathematical criteria (Montoya-Delgadillo & Reyes-Avendaño, 2022); suitable MWS, understood as the mode in which mathematical contents developed by a teacher or researcher are designed, adapted, and presented for teaching in a given place and context (Henríquez-Rivas et al., 2021, 2022); and personal MWS, linked to the reality of students' work when they appropriate and maneuver problem-solving (Menares-Espinoza & Vivier, 2022).

Research Questions

To evince how the accumulated progress and deepening of research on MWS has been achieved during the last few decades, the present study is oriented by the following research questions:

1. How are MWS studies distributed in terms of questions of methodology, such as authors' country of origin, year of publication, and type of document?
2. In the same sense as question one, how are MWS studies distributed in terms of research design, participants' education level, and data collection techniques?

Subsequently, the following questions are intended to delve deeper into contents of MWS-based research:

1. What research objectives are addressed in MWS-based studies?
2. Which theoretical and empirical aspects have been considered by researchers in their analyses?

Table 1. Inclusion & exclusion criteria

Inclusion criteria	Exclusion criteria
IC1. Studies at all levels of mathematics education.	EC1. Studies in a discipline other than mathematics education.
IC2. Studies addressing MWS in the theoretical framework or introduction.	EC2. Studies that do not address MWS in the results.
IC3. Studies published in English, French, and Spanish.	EC3. Studies published in languages other than English, French, or Spanish.
IC4. Document types: articles, conference proceedings, theses, and book chapters.	EC4. Types of studies: full books, book reviews, and notes.
IC5. Studies indexed in WOS, Scopus, SciElo, and Dialnet.	EC5. Studies indexed in databases other than those mentioned in IC5.

   Specifically, what mathematical domain has been favored in the studies, what types of MWS have been favored, and what characteristics are considered regarding tasks?

3. What links or connections have been made with other theories? What other theories or theoretical perspectives have been considered in research associated with MWS?

METHODOLOGY OF SYSTEMATIC REVIEW

Search Strategies & Selection Criteria

To determine the research trends in MWS theory, a systematic literature review was conducted, which is understood as "a review of existing research that uses rigorous, explicit, and responsible research methods" (Gough et al., 2012, p. 6). This literature review followed the guidelines stated by reporting items for systematic reviews and meta-analyses (PRISMA) protocol (Page et al., 2021) for documenting each stage of the review.

The final search for information was carried out on 9 August 2023 on Dialnet, SciElo, Scopus, and Web of Science databases. To identify relevant studies within the field of mathematics education, we employed the following search strings (for English): "mathematic working space" OR "mathematical working" OR "geometry working space" OR "probability working space". For French, search string was translated, and a query was executed across all four databases. For Spanish, searches were conducted in Dialnet and SciElo.

The search encompassed all documents with research on MWS theory, and restrictions were not established regarding education level or year of publication. Likewise, no automatic filters were applied in the corresponding databases. The inclusion and exclusion criteria are described in **Table 1**.

The selection process was carried out in three stages following the approach proposed by Page et al. (2021): identification, selection, and inclusion. In the identification stage, the search strings were utilized, returning a total of 453 articles. The results obtained from each database were imported to a Microsoft Excel spreadsheet and grouped to eliminate duplicates, leading to the exclusion of 158 duplicate results. In the selection phase, the titles, and abstracts of the remaining studies (295 studies) were reviewed, which led to 148 potentially relevant results being chosen (147 were not related to MWS). To finalize the selection stage, the inclusion and exclusion criteria were applied to the 148 studies in question. Lastly, in the final stage, 105 studies were included; however, it was not possible to obtain three of these documents, so ultimately the analysis included 102 studies. **Figure 2** shows a flowchart that summarizes the study selection process.

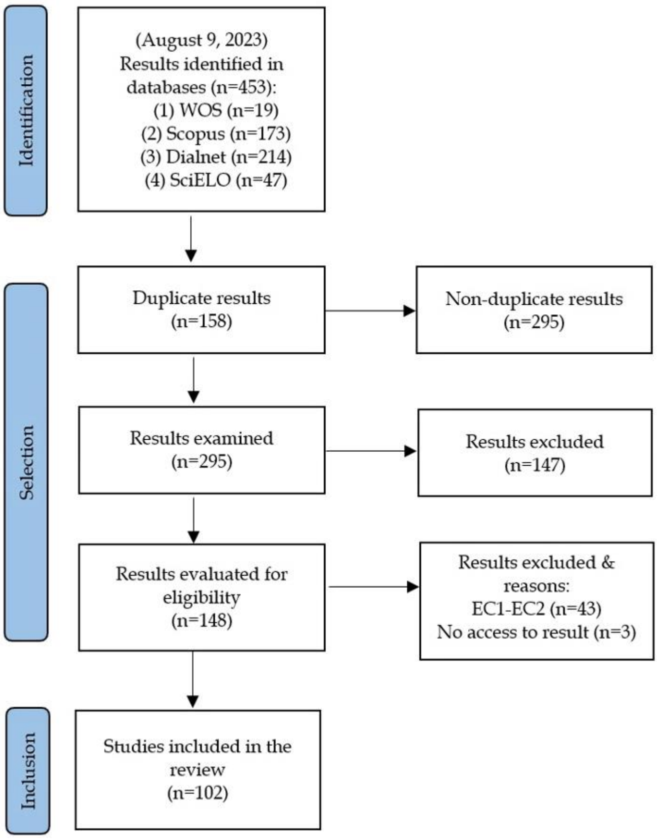

Figure 2. Diagram of process of identification & selection of studies (Source: Authors' own elaboration)

Data Analysis

A complete analysis of 102 studies was undertaken using a coding procedure established based on the

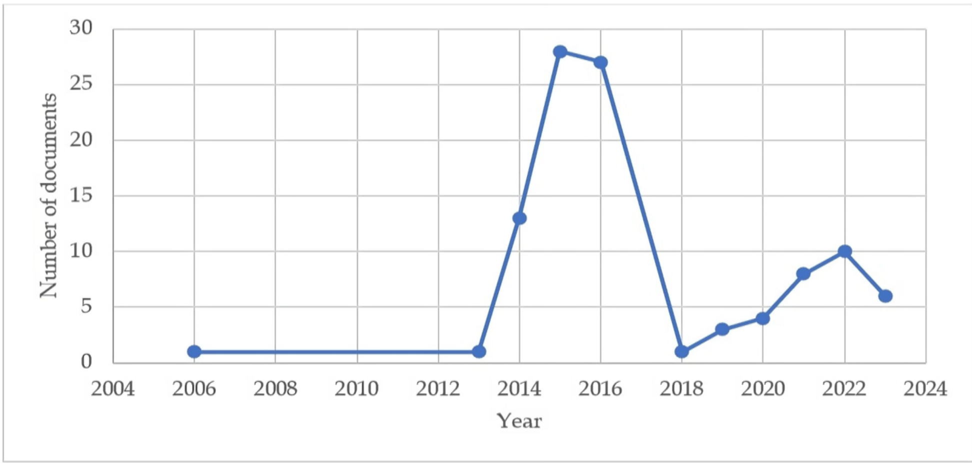

Figure 3. Distribution of studies based on year of publication (Source: Authors' own elaboration)

research questions. The studies in question were subject to exhaustive review through the technique of content analysis (Kuckartz & Rädiker, 2023).

During this process, methodological and theoretical elements characteristic of MWS theory were considered. In terms of methodological elements, the categories defined in the systematic review by Cevikbas et al. (2022), which is related to mathematical modeling skills, were examined; these include year of publication, type of document, geographical distribution of authors, research type and design, participants' education level, sample size, and data collection and analysis methods. Regarding theoretical categories, the factors analyzed included mathematical domain, types of MWS studied, the type of task addressed, and any linkage with other theories. This analysis provided an understanding of research on MWS theory and its diverse methodological and theoretical aspects.

RESULTS OF SYSTEMATIC REVIEW

Below, the results of the review of MWS theory in mathematics education are presented. They are organized into two groups in relation to the five research questions posed in the second subsection of the theoretical framework. The first group covers question 1 and question 2, which address methodological issues. The second group considers questions 3, 4, and 5, which address questions and objectives in MWS research, aspects related to the theoretical framework, and connections with other theoretical perspectives.

Analysis Group on Methodological Issues

First, general aspects are addressed, such as authors' country of origin, year or publication, and document type, along with certain methodological questions that aid in the understanding of how MWS theory has been researched and utilized in different contexts and education levels.

Document types of years of publication

This analysis included 102 studies of which 76 were empirical and 26 theoretical. These studies were distributed across different document types, as follows: 70 are journal articles, 29 are conference proceedings, two are book chapters, and one is a thesis. Among the articles, 53 are published in journals specialized in mathematics education, while five are published in science and mathematics education journals, six in education journals, five in interdisciplinary journals, and one in a technology journal. In terms of conference proceedings, all studies come from conferences on mathematics education. The majority originate from Symposium on Mathematical Work (n=26), while others are proceedings from International Group for the Psychology of Mathematics Education (PME) (n=2) and the conference organized by *Sociedad Española de Investigación en Educación Matemática* [*Spanish Society for Research in Mathematics Education*] (SEIEM) (n=1).

Regarding the years of publication, the studies reviewed were published between 2006 and 2023, as shown in **Figure 3**. It is important to highlight that in 2014, *Revista Latinoamericana de Investigación en Matemática* [*Latin American Journal of Research in Mathematics*] (RELIME), dedicated two issues to research on MWS theory. Likewise, this occurred with journals *Bolema* (Boletim de Educação Matemática [Mathematics Education Bulletin]) and *ZDM-Mathematic Education* in 2016. Meanwhile, in 2015, proceedings of 4th Symposium on MWS Theory were registered.

Geographical distribution of authors

Authors who have utilized MWS theory are located in different parts of the world, and their institutional affiliations reflect a wide geographic distribution (**Figure 4**). Examining the affiliations of all authors recorded in the publications analyzed reveals that they originate from a total of 15 countries, eight of which are from the Americas and seven from Europe. The countries with greatest representation in the publications reviewed are Chile (23.0%), Spain (21.0%), France (16.7%), and Mexico

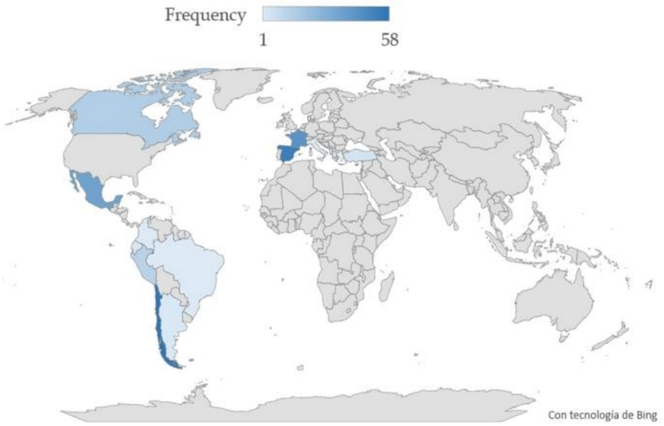

Figure 4. Distribution of authors by country (Source: Authors' own elaboration)

(14.3%). The other countries recorded comprise less than 7.0% of the total. It should be noted that the results show that 81.0% of the articles analyzed have less than three authors, while the maximum number of authors was six.

Research design & education level of study participants

The analysis showed that, of the 76 empirical studies, 69 correspond to qualitative research, and three to quantitative research, while four combine both qualitative and quantitative methods, but without explicitly being mixed-methods research. Meanwhile, 26 of the studies reviewed are theoretical studies. In terms of the methods utilized most frequently in empirical studies, 24.0% employed case study research, while 6.0% used didactic engineering as a methodological approach.

In terms of participants in the studies analyzed, four studies involve primary school students, 25 involve secondary school students, eight involve university students from non-teaching-focused programs (e.g., mathematics or engineering majors), 15 involve pre-service mathematics teachers (four primary and 11 secondary), 16 involve in-service mathematics teachers (four primary and 12 secondary), and eight utilize mixed samples of primary and secondary school students, university students, and pre-service mathematics teachers. This distribution is illustrated in **Figure 5**.

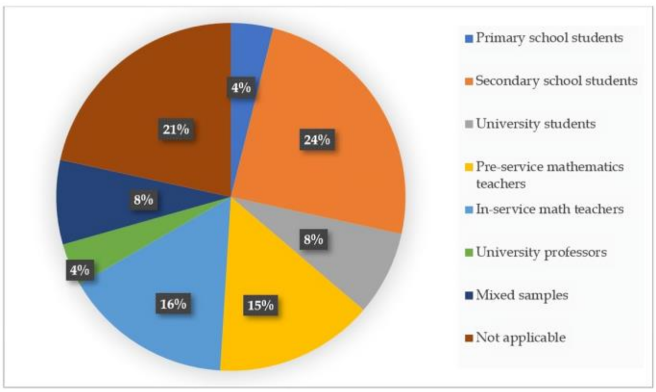

Figure 5. Education level of participants in studies analyzed (Source: Authors' own elaboration)

Table 2. Sample size/number of participants reported in studies

Sample size/participants	Frequency (n)	Percentage (%)
0-50	52	51.0
51-100	5	4.9
101-200	1	1.0
201-500	5	4.9
>500	3	2.9
Not mentioned	11	10.8
Not applicable	25	24.5
Total	102	100

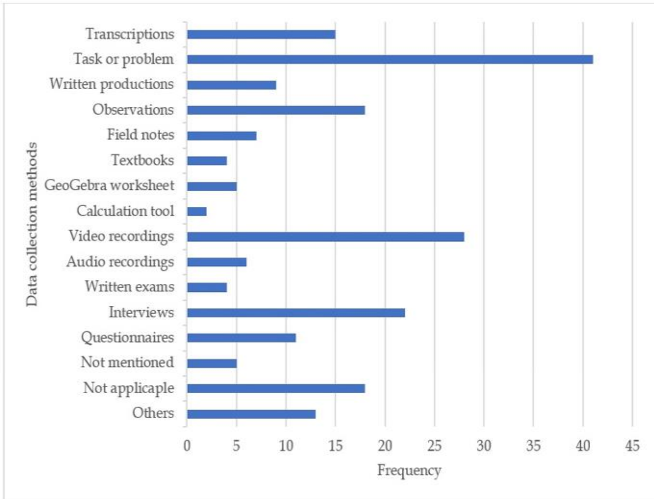

Figure 6. Data collection methods (Source: Authors' own elaboration)

Table 2 shows the sample sizes (or numbers of participants) utilized in the studies analyzed. Half of these studies (n=52) use samples with fewer than 50 participants, 11 studies do not mention sample size, and five studies use sample sizes that range from 51 to 100 participants; meanwhile, five studies use samples ranging from 201 to 500 participants.

Data collection methods

The analysis revealed that 79 of the 102 studies utilize a range of one to seven distinct methods of data collection. More specifically, 83.5% of the studies employ between one and three data collection methods. The most used method, employed in 40.2% (n=41) of studies, is the application of tasks or problems, while 27.5% (n=28) rely on video recordings, and 21.6% (n=22) utilize interviews. The remainder of methods reported are used in less than 10.0% of studies, as shown in **Figure 6**.

Analysis Groups on Theoretical Topics

Below, results are presented in relation to the objectives proposed by the studies, theoretical aspects specific to MWS, and connections between MWS and other theoretical perspectives. In particular, in terms of theoretical topics, attention is focused on mathematical domains, tasks, and types of MWS.

Table 3. Objectives in MWS investigations

Objectives addressed	Frequency (n)	Percentage (%)
Analyze	21	20.6
Approximate	1	1.0
Characterize	14	13.7
Complement	1	1.0
Understand	6	5.9
Know	2	2.0
Describe	2	2.0
Discuss	1	1.0
Study	3	2.9
Evaluate	1	1.0
Examine	2	2.0
Explore	6	5.9
Identify	3	2.9
Investigate	1	1.0
Demonstrate	3	2.9
Compare	2	2.0
Specify	1	1.0
Expound	1	1.0
Propose	1	1.0
Relate	1	1.0
Not made explicit	29	28.4
Total	102	100

Research objective addressed by studies

The findings demonstrate that the most frequent research objectives include the following purposes: analyzing (n=21), characterizing (n=14), exploring (n=6), and understanding (n=6).

The studies centered on analyzing tend to address diverse forms of solving a task or problem, examine the work of an expert solving a problem in the context of referential MWS, or analyze MWS when students or teachers solve a task. Meanwhile, those studies that seek to characterize specifically describe the mathematical work, either of teachers or students, when solving a task or problem. Regarding the studies with objectives focused on understanding, the majority seek to understand a mathematical concept through MWS theory. A synthesis of the objectives across all studies analyzed is presented in **Table 3**.

Mathematical domain

In this review, the mathematical domains studied based on MWS theory have also been examined. **Table 4** shows that 65.0% of the studies analyzed are focused on one mathematical domain: 31.4% of studies are centered on the domain of geometry, nearly 5.9% on algebra, and approximately 9.8% on arithmetic and analysis, respectively, while fewer studies explore statistics, and probability. Meanwhile, approximately 19.0% of studies analyzed are concerned with two mathematical domains in tandem; the pairs of domains most studied are algebra and arithmetic (n=3), algebra and geometry (n=3), and arithmetic and geometry (n=4). Of the 19 studies that analyze two domains, only 12 explicitly address the change from one domain to another in their analysis.

Table 4. Domains analyzed in studies

Mathematical domains	Frequency (n)	Percentage (%)
Algebra	6	5.9
Analysis	10	9.8
Arithmetic	10	9.8
Geometry	32	31.4
Probability	4	3.9
Statistics	3	2.9
Mixed	19	18.6
Not applicable	15	14.7
Not mentioned	3	2.9
Total	102	100

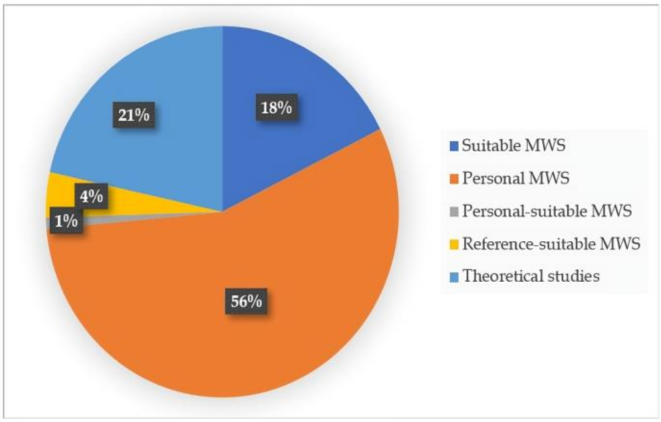

Figure 7. Types of MWS reported in studies (Source: Authors' own elaboration)

Types of mathematical working spaces & tasks

Regarding types of MWS, 57 studies were found to focus on the personal MWS of the participants of which 54 are empirical studies and three are theoretical studies (**Figure 7**). Meanwhile, the suitable MWS is present in 18 studies, with 17 corresponding to empirical studies and one to a theoretical study. Some studies analyze two types of MWS, including personal-suitable (n=1) and referential-suitable (n=4). No theoretical and empirical studies were found that analyze all MWS types. Moreover, 22 of the studies are theoretical in nature and do not address any specific type of MWS.

Meanwhile, in terms of types of MWS and mathematical domains, the personal MWS is utilized most to analyze the geometric domain (n=19) and the arithmetic domain (n=10) in the studies reviewed. The other domains are represented in three studies or less in relation to this type of MWS. Suitable MWS has been studied in relation to the geometric domain (n=6) and in relation to other domains to a lesser extent, with three studies or fewer.

Additionally, of the 12 studies that analyze a *change of domain*, 10 are of empirical nature. Two are centered on the suitable MWS and eight on the personal MWS, while two are theoretical studies.

Table 5. Task types reported in studies

Task types	Frequency (n)	Percentage (%)
Adapted	2	2.0
Unpublished	48	47.1
From prior research	8	7.8
From textbook	5	4.9
Proposed by teacher	13	12.7
Various	2	2.0
Not mentioned	3	2.9
Not appliable	21	20.6
Total	102	100

Mathematical tasks

In this review, the task types utilized in studies related to MWS theory were also analyzed. 48 studies employed unpublished tasks, which means that the authors designed their own tasks considering their specific studies and taking their research objective into account. In 13 studies, the tasks were proposed by the teacher in the context of teaching with examples, class activities, or evaluations. In eight studies, the tasks were drawn from existing research, which implies that tasks were utilized that had already been developed in other publications. In five studies, tasks from textbooks were used, both K-12 and university level. Furthermore, two studies combine the categories mentioned above. This data is exhibited in **Table 5**.

Links/connections with other theories or perspectives

During the review, several studies were found in which MWS theory is linked to other theories or perspectives in mathematics education research. Those, which were most often utilized in connection to MWS to analyze data included mathematical modeling (n=9), mathematics teachers' specialized knowledge model (MTSK) (n=8), activity theory (n=4), affect in mathematics education (n=3), and problem-solving (n=3). Some studies also address other theories of mathematics education, but in lesser measure. This data is summarized in **Table 6**.

Table 6. Theories linked with MWS in studies

Theories linked with MWS	Frequency (n)	Percentage (%)
Affect in mathematics education	3	2.9
Documentational approach to didactics	1	1.0
Statistical inquiry cycle	1	1.0
Mathematical modeling	9	8.8
MTSK	8	7.8
Problem-solving	3	2.9
Anthropological theory of didactics	1	1.0
Activity theory	4	3.9
Socio-epistemological theory	1	1.0
Not considered	71	69.6
Total	102	100

CONCLUSIONS

This systematic review of the current state of research underpinned by MWS theory is based on the analysis of 102 studies, including empirical and theoretical research, indexed in databases including Dialnet, SciElo, Scopus, and Web of Science. The main purpose of the study is to explore the characteristics of existing MWS research, including basic factors such as authors' country of origin, year of publication, and type of study; furthermore, this study analyzes relevant methodological factors in relation to research design, participant characteristics, and data collection techniques. Other, more specific aspects of the studies analyzed are also considered in this review, such as stated research objectives, mathematical domain researched, types of MWS, and characteristics of studies regarding tasks. Lastly, connections with other theories present in MWS research are also reported.

Regarding the first research question, the results indicate that the majority of authors of the studies reviewed are from Chile, Spain, France, or Mexico. In this sense, it should be mentioned that participation in joint international projects (e.g., French-Chilean ECOS-Sud C13H03 project) have contributed to developing and deepening understanding of important theoretical aspects of MWS (e.g., Montoya-Delgadillo & Vivier, 2016). Also, proceedings of MWS Symposium and issues dedicated to theory in prestigious mathematics education journals have had a strong impact on number of documents found and analyzed in this review.

From a methodological perspective, the results reveal that nearly 26.0% of studies analyzed focus on theoretical aspects of MWS, and 69.0% are carried out in qualitative research frameworks. It is important to highlight that a large quantity of these studies is based on case study research. Meanwhile, the majority of participants were secondary school students and in-service mathematics teachers. In terms of data collection techniques, the most common were the application of tasks or problems and video recordings.

The results suggest that an opportunity exists for future studies to consider qualitative research focused on education levels including early childhood, primary, and adult education, where mathematical work can be strengthened based on teacher education and the efforts of teacher educators. Meanwhile, quantitative research approaches have been less utilized from this theoretical perspective, and they could be considered, for example, in mixed-methods research or projects (Creswell & Creswell, 2023) in which MWS theory could contribute not only to describing or characterizing mathematical work, but also to understanding and transforming MWS of diverse actors (teachers, students, investigators) in an educational context (Kuzniak et al., 2022). The latter point in particular (transforming MWS) stands out as a key opportunity for future research.

In relation to types of research objectives addressed in the scope of MWS theory, *analyzing* and *characterizing* are the most frequent. This underscores the diversity of analysis that has been carried out sustained by MWS theory over time. Findings illustrate a wide panorama of the advances made in this field and offer orientation toward future research that seeks to delve deeper into the process of teaching and learning mathematics through MWS theory. Furthermore, this theory has allowed for analytical and methodological models to be proposed for analyzing mathematical work based on distinct perspectives, levels, contexts, and participants.

In terms of specific aspects of the studies reviewed, one important point is that the domains that have been favored thus far are geometry and arithmetic. These findings point to the necessity of considering studies in other mathematical domains, as well as greater focus on changes of domain (Montoya-Delgadillo & Vivier, 2014). Regarding types of MWS, it is evident that referential MWS has been the least studied, which opens possibilities for study and exploration from a theoretical and methodological point of view. In this sense, future research that focuses on the referential MWS of teachers at different education levels could contribute to the formulation of improvements in mathematics teacher education in terms of curriculum, teaching organization, and the design of signature tasks and materials available for students and teachers (Kuzniak, 2022), all of which could fall within the purview of researchers utilizing this perspective. As observed in **Table 5**, 31 of the 102 studies reviewed were developed in connection with other theories. Of these 31 studies, it is noteworthy that nine utilize mathematical modeling, while eight also consider MTSK model (Carrillo et al., 2015). Some of these examples are developed explicitly from the perspective of networking theories (e.g., Verdugo-Hernández et al., 2022). Without a doubt, this research perspective merits further exploration.

Concerning the limitations or restrictions of the present study, it is important to mention that the exclusion criteria of certain databases may have resulted in the omission of studies relevant to MWS theory; for example, only one doctoral thesis based on MWS theory was considered, and the proceedings of conferences such as RELME or CERME have not been accounted for. Moreover, the lack of access to some documents could have limited the inclusion of certain studies in the analysis. In future research, manual searches could be carried out that consider other search terms or the detection of studies published in journals whose indexation is distinct from that considered in the present review. But this systematic review largely reflects advances and developments that MWS theory has undergone both in recent years and since its beginnings.

Ultimately, this study presents a systematic review of the current state of existing literature built upon the framework of MWS theory, demonstrating an expansive body of research that could contribute to further research in this theoretical line.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: This study was funded by the Beca Doctoral Universidad Católica de Maule 2023 & Agencia Nacional de Investigación y Desarrollo de Chile, Fondo Nacional de Desarrollo Científico y Tecnológico de Iniciación 2023, Folio 11230523.

Ethical statement: The authors stated that ethics committee approval for this systematic review is not required.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. (2015). *Approaches to qualitative research in mathematics education: Examples of methodology and methods*. Springer. https://doi.org/10.1007/978-94-017-9181-6

Carrillo, J., Flores-Medrano, E., Contreras, L. C., & Climent, N. (2015). El profesor en el marco de los ETM el papel del MTSK como modelo de conocimiento [The teacher in the framework of the ETM the role of the MTSK as a model of knowledge]. In I. M. Gómez-Chacón, J. Escribano, A. Kuzniak, & P. R. Richard (Eds.), *Proceedings of the 4th ETM Symposium* (pp. 459-469). Universidad Complutense de Madrid.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics, 109*(2), 205-236. https://doi.org/10.1007/s10649-021-10104-6

Coutat, S., & Richard, P. (2011). Les figures dynamiques dans un espace de travail mathématique pour l'apprentissage des propriétés géométriques [Dynamic figures in a mathematical workspace for learning geometric properties]. *Annales de Didactique et de Sciences Cognitives* [*Annals of Didactics and Cognitive Sciences*], *16*, 97-126.

Creswell, J. W., & Creswell, J. D. (2023). *Research design. Qualitative, quantitative and mixed methods approches* (6th Edn.). SAGE.

Derouet, C., & Parzysz, B. (2016). How can histograms be useful for introducing continuous probability distributions? *ZDM Mathematics Education, 48*(6), 757-773. https://doi.org/10.1007/s11858-016-0769-9

Derouet, C., Nechache, A., Richard, P. R., Vivier, L., Gómez-Chacón, I. M., Kuzniak, A., Maschietto, M., & Montoya-Delgadillo, E. (2023). *Actes du Septième Symposium d'Étude sur le Travail Mathématique* [*Proceedings of the Seventh Mathematical Work Study Symposium*]. IREM de Strasbourg.

Gaona, J., & Arévalo-Meneses, F. (2023). *Análisis bibliométrico temático de 37 revistas especializadas en investigación en educación matemática indexadas en Scopus y Web of Science* [*Thematic bibliometric analysis of 37 journals specialized in mathematics education research indexed in Scopus and Web of Science*]. https://osf.io/preprints/edarxiv/xraz6

Gómez-Chacón, I. M., Botana, F., Escribano, J., & Abánades, M. (2016a). Concepto de lugar geométrico. Génesis de utilización personal y profesional con distintas herramientas [Geometric locus concept. Genesis of personal and professional use with different tools]. *Bolema* [*Bulletin*], *30*(54), 67-94. https://doi.org/10.1590/1980-4415v30n54a04

Gómez-Chacón, I. M., Kuzniak, A., & Vivier, L. (2016b). El rol del profesor desde la perspectiva de los espacios de trabajo matemático [The role of the teacher from the perspective of mathematical work spaces]. *Bolema* [*Bulletin*], *30*(54), 1-22. https://doi.org/10.1590/1980-4415v30n54a01

Gómez-Chacón, I. M., Romero Albaladejo, I. M., & del Mar García López, M. (2016c). Zigzagging in geometrical reasoning in technological collaborative environments: A mathematical working space-framed study concerning cognition and affect. *ZDM Mathematics Education, 48*(6), 909-924. https://doi.org/10.1007/s11858-016-0755-2

Gómez-Chacón, I. M., Escribano, J., Kuzniak, A., & Richard, P. (2015). *Proceedings of the 4th ETM Symposium.* Universidad Complutense de Madrid.

Gómez-Chacón, I. M., Kuzniak, A., Nikolantonakis, K., Richard, P., & Vivier, L. (2017). *Proceedings of the 5th ETM Symposium.* University of Western Macedonia.

Gough, D., Oliver, S., & Thomas, J. (2012). *An introduction to systematic reviews*. SAGE.

Henríquez-Rivas, C., & Kuzniak, A. (2021). Profundización en el trabajo geométrico de futuros profesores en entornos tecnológicos y de lápiz y papel [Deepening the geometric work of future teachers in technological and pencil and paper environments]. *Bolema*, *35*(71), 15501572. https://doi.org/10.1590/1980-4415V35N71A15

Henríquez-Rivas, C., & Montoya-Delgadillo, E. (2015). Espacios de trabajo geométrico sintético y analítico de profesores y su práctica en el aula [Synthetic and analytical geometric workspaces of teachers and their practice in the classroom]. *Enseñanza de las Ciencias, 33*(2), 5170. https://doi.org/10.5565/rev/ensciencias.1408

Henríquez-Rivas, C., & Montoya-Delgadillo, E. (2016). El trabajo matemático de profesores en el tránsito de la geometría sintética a la analítica en el liceo [The mathematical work of teachers in the transition from synthetic geometry to analytics in high school]. *Bolema*, *30*(54), 4566. https://doi.org/10.1590/1980-4415v30n54a03

Henríquez-Rivas, C., Guerrero-Ortiz, C., & Barrera, A. (2021). Trabajo matemático de profesores universitarios: Heurísticas de solución de una tarea [Mathematical work of university professors: Heuristics for solving a task]. E*ducación Matemática*, *33*(3), 233262. https://doi.org/10.24844/EM3303.09

Henríquez-Rivas, C., Kuzniak, A., & Masselin, B. (2022). The idoine or suitable MWS as an essential transition stage between personal and reference mathematical work. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of Mathematical Working Spaces* (pp. 121-146). Springer. https://doi.org/10.1007/978-3-030-90850-8_6

Henríquez-Rivas, C., Ponce, R., Carrillo, J., Climent, N., & Espinoza-Vásquez, G. (2021). Trabajo matemático de un profesor basado en tareas y ejemplos propuestos para la enseñanza [Mathematical work of a teacher based on tasks and examples proposed for teaching]. *Enseñanza de Las Ciencias*, *39*(2), 123142. https://doi.org/10.5565/rev/ensciencias.3210

Houdement, C., & Kuzniak, A. (1996). Autours des stratégies utilisées pour former les maîtres du premier degré en mathématiques [Around the strategies used to train primary school teachers in mathematics]. *Recherches en Didactique des Mathématiques, 16*(3), 283-312.

Houdement, C., & Kuzniak, A. (1999). Un exemple de cadre conceptuel pour de l'enseignement de la géométrie en formation des maîtres [An example of a conceptual framework for teaching geometry in teacher training]. *Educational Studies in Mathematics, 40,* 283-312. https://doi.org/10.1023/A:1003851228212

Kuckartz, U., & Rädiker, S. (2023). *Qualitative content analysis: Methods, practice and software*. SAGE. https://doi.org/10.4324/9781003213277-57

Kuzniak, A. (2006). Paradigmes et espaces de travail géométriques. Éléments d'un cadre théorique pour l'enseignement et la formation des enseignants en géométrie [Paradigms and geometric workspaces. Elements of a theoretical framework for teaching and teacher training in geometry]. *Canadian Journal of Science, Mathematics and Technology Education*, *6*(2), 167187. https://doi.org/10.1080/14926150609556694

Kuzniak, A. (2011). L'Espace de Travail Mathematique et ses genèses [The mathematical workspace and its genesis]. *Annales de Didactique et de Sciences Cognitives*, *16*, 9-24.

Kuzniak, A. (2014). Travail mathématique et domaines mathématiques [Mathematical work and mathematical domains]. *Revista Latinoamericana de Investigación En Matemática Educativa, 17*(4II), 385399. https://doi.org/10.12802/relime.13.17419

Kuzniak, A. (2018). Thinking about the teaching of geometry through the lens of the theory of geometric working spaces. In P. Herbst, U. Cheah, P. Richard, & K. Jones (Eds.), *International Perspectives on the Teaching and Learning of Geometry in Secondary Schools. ICME-13 Monographs* (pp. 5-21). Springer. https://doi.org/10.1007/978-3-319-77476-3_2

Kuzniak, A. (2022). The theory of mathematical working spaces theoretical characteristics. In A. Kuzniak, E. Montoya-Delgadillo, & P. Richard. *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 3-31). Springer. https://doi.org/10.1007/978-3-030-90850-8_1

Kuzniak, A., & Nechache, A. (2021). On forms of geometric work: A study with pre-service teachers based on the theory of Mathematical Working Spaces. *Educational Studies in Mathematics*, *106*(2), 271289. https://doi.org/10.1007/s10649-020-10011-2

Kuzniak, A., & Richard R., P. (2014). Espacios de trabajo matemático. Puntos de vista y perspectivas [Mathematical workspaces. Points of view and perspectives]. *Revista Latinoamericana de Investigación En Matemática Educativa*, 17(4I), 515. https://doi.org/10.12802/relime.13.1741a

Kuzniak, A., Montoya-Delgadillo, E., & Richard, P. (2022). *Mathematical work in educational context: The perspective of the theory of mathematical working spaces*. Springer. https://doi.org/10.1007/978-3-030-90850-8

Kuzniak, A., Nechache, A., & Drouhard, J. P. (2016a). Understanding the development of mathematical work in the context of the classroom. *ZDM Mathematics Education*, *48*(6), 861874. https://doi.org/10.1007/s11858-016-0773-0

Kuzniak, A., Tanguay, D., & Elia, I. (2016b). Mathematical working spaces in schooling: An introduction. *ZDM - Mathematics Education*, *48*(6), 721737. https://doi.org/10.1007/s11858-016-0812-x

Laborde, C. (2007). Towards theoretical foundations of mathematics education. *ZDM Mathematics Education, 39*, 137-144. https://doi.org/10.1007/s11858-006-0015-y

Lerman, S. (2020). *Encyclopedia of mathematics education*. Springer. https://doi.org/10.1007/978-3-030-15789-0

Menares Espinoza, R., & Vivier, L. (2022). Personal mathematical work and personal MWS. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 91-120). Springer. https://doi.org/10.1007/978-3-030-90850-8_5

Montoya-Delgadillo, E., & Reyes-Avendaño, C. (2022). The reference mathematical working space. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 73-90). Springer. https://doi.org/10.1007/978-3-030-90850-8_4

Montoya-Delgadillo, E., Richard, P., Vivier, L., Gómez-Chacón, I., Kuzniak, A., Maschietto, M., & Tanguay, D. [Eds.] (2019). *Sexto Simposio sobre el Trabajo Matemático* [Sixth Symposium on Mathematical Work]. Pontificia Universidad Católica de Valparaíso, Facultad de Ciencias. https://www.pucv.cl/uuaa/site/docs/20200102/20200102160133/actes_etm6.pdf

Montoya-Delgadillo, E., & Vivier, L. (2014). Les changements de domaine de travail dans le cadre des espaces de travail mathématique [Changes in work domain within the framework of mathematical workspaces]. *Annales de Didactique et de Sciences Cognitives* [*Annals of Didactics and Cognitive Sciences*], *19*, 73-101.

Montoya-Delgadillo, E., & Vivier, L. (2016). Mathematical working space and paradigms as an analysis tool for the teaching and learning of analysis. *ZDM Mathematics Education, 48*(6), 739-754. https://doi.org/10.1007/s11858-016-0777-9

Montoya-Delgadillo, E., Mena-Lorca, A., & Mena-Lorca, J. (2014). Circulaciones y génesis en el espacio de trabajo matemático [Circulations and genesis in the mathematical workspace]. *Revista Latinoamericana de Investigación en Matemática Educativa* [*Latin American Journal of Research in Educational Mathematics*], *17*(4-I), 81-197. https://doi.org/10.12802/relime.13.1749

Nechache, A., & Gómez-Chacón, I. M. (2022). Methodological aspects in the theory of mathematical working spaces. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context* (pp. 33-56). Springer. https://doi.org/10.1007/978-3-030-90850-8_2

Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., …, & Moher, D. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *The BMJ, 372*(71). https://doi.org/10.1136/bmj.n71

Radford, L. (2017). On inferentialism. *Mathematics Education Research Journal, 29*(4), 493-508. https://doi.org/10.1007/s13394-017-0225-3

Schoenherr, J., & Schukajlow, S. (2023). Characterizing external visualization in mathematics education research: A scoping review. *ZDM Mathematics Education, 56*(1), 73-85. https://doi.org/10.1007/s11858-023-01494-3

Vásquez, C., Alsina, A., Seckel, M. J., & García-Alonso, I. (2023). Integrating sustainability in mathematics education and statistics education: A systematic review. *EURASIA Journal of Mathematics, Science and Technology Education, 19*(11), em2357. https://doi.org/10.29333/ejmste/13809

Verdugo-Hernández, P., Espinoza-Vásquez, G., & Yáñez, J. C. (2022). Análisis de una tarea sobre sucesiones desde el uso de las herramientas y el conocimiento matemático del professor [Analysis of a task on sequences from the use of tools and the teacher’s mathematical knowledge]. *Enseñanza de Las Ciencias* [*Science Teaching*], *40*(2), 125-145. https://doi.org/10.5565/rev/ensciencias.3457

https://www.ejmste.com