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Lessons Learnt from Employing van Hiele Theory Based Instruction in Senior Secondary School Geometry Classrooms

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This paper reports on a part of a study which was conducted to determine the effect of van Hiele theory based instruction in the teaching of geometry to Grade 10 learners. The sample consisted of 359 participants from five conveniently selected schools from Mthatha District in the Eastern Cape Province in South Africa. There were 195 learners in the experimental group and 164 learners in the control group. The experimental group was given van Hiele theory based geometry instruction and the control group was given traditional method of geometry instruction. A multiple choice geometry test was administered to the participants before and after five weeks of instruction (pre- and post-test design). The results indicated a statistically significant difference in the mean scores in favour of the experimental group. The significant improvement in the performance of the experimental group having more learners at level 2 than at level 0 and level 1 suggest that the van Hiele-based instruction had a positive effect in raising the learners' levels of thinking compared to traditional instruction.

Keywords: geometry, learners' levels of thinking, van Hiele theory based instruction

INTRODUCTION

Geometry is an essential part of the mathematics curriculum in the South African senior secondary schools. In the 90s, Snyders (1995) observed that geometry is regarded as a problematic branch of mathematics around the world. Although research on learning and teaching geometry can be traced back to a few decades, recent research shows that the interest in the subject has remained topical (Abdullah & Zakaria, 2011; De Villiers, 2010; Erdogan, Akkaya & Celebi Akkaya, 2009; Gujarati,

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2014; Haviger & Vojkůvková, 2014; Howse & Howse, 2014; Pittalis, Mousoulides & Christou, 2009; Siew, Chong & Abdullah, 2013; Yegambaram & Naidoo, 2010).

Geometry is connected to every strand in the mathematics curriculum and to a multitude of situations in real life (Yegambaram & Naidoo, 2010). Spatial understanding is necessary for interpreting and appreciating our inherently geometric world (NCTM, 1989). Geometry is an important component of mathematics and is required for students to better understand facts about the world (Erdogan et al., 2009*)*. Knowledge of geometry remains a pre-requisite in fields such as "physics, astronomy, art, mechanical drawing, chemistry (for atomic and molecular structure), biology (for cell structure), and geology (for crystalline structure)" (Sherard, 1981, p.20). Geometric skills are important in architecture and design, in engineering, and in various aspects of construction work. The fields of study mentioned here play a major role in the development of any given country. Geometry is the mathematics of space and mathematicians search for mathematical interpretations of space (Bishop, 1983). School geometry is the study of those spatial objects, relationships and transformations that have been mathematised, and the axiomatic mathematical systems that have been constructed to represent them (Clements & Battista, 1992). In a sense geometry focuses on the development and application of spatial concepts through which children learn to represent and make sense of the world (Thompson, 2003). It is for these reasons that South African learners are compelled to study geometry. This is in order to avail a wide range of options from which they can choose appropriate occupations.

Malloy (1999) states that historically, understanding geometric concepts and also developing and reproducing proofs have been problematic for many teachers. The authors conclude that both students and teachers consider geometry to be the most dreaded topic in high school mathematics. Researchers (e.g., Clements & Battista, 1992; De Villiers, 1996; Hoffer, 1981; Senk, 1985; Shaughnessy & Burger, 1985; Siyepu, 2005) have documented that high school learners are not ready for formal proofs in the senior secondary schools and stressed the need for more informal geometry instruction in junior secondary schools. Senior secondary school learners experience difficulties in understanding terminology in plane geometry, identifying and classifying shapes, properties of shapes and proof writing (Atebe, 2008; Clements & Battista, 1992; Fuys, Geddes & Tischler, 1988; Siyepu, 2005; Usiskin, 1982). According to Murray (1997), the geometry studied in the primary school has traditionally been in preparation for senior schools.

Understanding geometry has been an area of research over the past six decades. One of the models proposed in the 1950's was the theoretical perspective that was put forward by two Dutch mathematicians, Pierre van Hiele and his wife

State of the literature

- The van Hiele theory of geometrical thinking is aimed at improving teaching by organising instruction to take into account learner's levels of thinking.
- To improve geometry teaching, educators need to develop tasks or activities that help them better understand the nature of their learners' geometric reasoning.
- In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum and carrying out research based on van Hiele theory in rural South African schools is pertinent as the poor performance of learners in geometry in South Africa has been a topic of concern over the past four decades.

Contribution of this paper to the literature

- In a revision of the curriculum in South Africa in 2012, Euclidean geometry was re-introduced in senior secondary schools.
- This study addressed the deficiencies in senior secondary school geometry instruction in South Africa by identifying the learners' levels of understanding in geometry first and used that knowledge in developing an instructional framework based on the van Hiele theory to enhance the geometry instruction.
- The study developed activities that could be implemented without expensive or sophisticated materials. The results from the study confirmed that higher levels of geometric thinking can be attained by the implementation of educator guided, learner centered and hands-on instructional programme.

Dina van Hiele-Geldof. A feature of their model is a five level hierarchy. Each of the levels (levels 1 – 5) describes the thinking processes used in geometric contexts (van de Walle, 2001). According to Shulman (1987), teaching begins with a teacher's understanding of what is to be learned and how it is to be taught. This author places emphasis on pedagogical content knowledge (PCK) as one of the main categories of the knowledge base of teachers which tunes content to achieve pedagogical outcomes successfully (Shulman, 1987). According to Rossouw and Smith (1997), the rich base developed from research on van Hiele levels and how students learn geometry is an important source to understanding teachers' PCK on geometry teaching. The teachers' geometrical PCK is a distinctive knowledge that is needed in order to transform the content to be interesting and comprehensible to the learners (Rossouw & Smith, 1997).

The question posed in this paper was: Can an instructional strategy following the van Hiele levels improve geometric thinking of learners?

GEOMETRY INSTRUCTION IN THE SOUTH AFRICAN CONTEXT

Despite geometry being an important branch of mathematics, there are many challenges in learning and teaching it. According to King (2003), dissatisfaction with the secondary school geometry curriculum and the poor performance of learners in geometry in South Africa has been a topic of concern over the past four decades. In 1997, the Geometry working Group of a South African Non-Governmental Organisation called Mathematics Learning and Teaching Initiative (MALATI) tried to re-conceptualise the teaching and learning of geometry (Bennie, 1998). For that re-conceptualisation to happen and to propose changes to the curriculum, the group felt that a means to understand the geometric thinking of learners would be needed (King, 2003). The group found that the van Hiele model of geometric thinking could be used as a framework to understand the geometric thinking of learners. The idea of re-conceptualising the approach to geometry teaching and learning was placed in the foreground of the introduction of Curriculum 2005 in 1998 in South Africa (King, 2003). Breen (1997) also indicated that in South African primary schools the geometry instruction was insufficient in terms of providing learners with the necessary skills to function at the level of axiomatic thinking in senior secondary schools. De Villiers (1997) suggested that a revision of the primary school geometry curriculum along the van Hiele levels would ensure success in the senior secondary school.

De Villiers (2010) states that formal geometry curriculum is focussed on senior secondary school education while there is relatively little content in the primary school. Although tessellations are recently introduced in the primary school, many teachers and textbook authors do not appear to understand its relevance in relation to the van Hiele theory (De Villiers, 2010). Feza and Webb (2005) indicate that the South African National Curriculum Statement (NCS) at the intermediate phase reflects levels 1, 2 and 3 in the van Hiele hierarchy. This is because learners are required to describe and represent the characteristics and relationships between two-dimensional and three-dimensional objects in a variety of orientations and positions (Feza & Webb, 2005). In a revision of the curriculum in South Africa in 2012 Euclidean geometry was re-introduced. In the curriculum revision known as the Curriculum and Assessment Policy Statement (CAPS), in Grade 10 learners are expected to (a) investigate and form conjectures about the properties of special triangles, quadrilaterals and other polygons. They need to try to validate or prove conjectures using any logical method (Euclidean, coordinate or transformation geometry from Grade 9), disapprove false conjectures by producing counter examples and (b) investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangles, the kite, parallelogram, rectangle, rhombus and square) (South African Department of Basic Education, 2011).

Atebe and Schafer (2009) stated that the teaching and learning of geometry is one of the most disappointing experiences in many schools across nations. In a study conducted in underperforming schools in South Africa, Mji and Makgato (2006, p.261) reported on factors affecting the poor performance in mathematics and one of the responses from participants was "…we spend most of the time learning algebra which is easy but what about geometry which is difficult? That is why we do little geometry…" Mji and Makgato (2006) state that outdated teaching practices and lack of basic content knowledge have resulted in poor teaching standards. Also, the process of teaching and learning at the secondary school level (traditional method), which gives less opportunity to learn at the students' own pace is a potential factor hindering students' achievement in mathematics. The prevailing learning activities in schools consisting of mainly listening, watching and imitating the teacher are not supportive of efficient learning in mathematics (Akinsola & Ifamuyiwa, 2008).

THE VAN HIELE THEORY

The Van Hiele Model was created to provide geometric understanding and to develop geometric understanding (Erdogan et al, 2009*)*. This model provides useful empirically-based descriptions of what are likely to be relatively stable and qualitatively different states of understanding in learners (Ding & Jones, 2007). According to Malloy (2002), Pierre van Hiele & his wife, Dina van Hiele – Geldof, having been concerned with the difficulties their students encountered with secondary school geometry, began to think that the content they were teaching was too advanced for many of their students to fully understand. The van Hiele couple believed that secondary school geometry involved thinking at a relatively "higher level" and their students did not have sufficient experiences in thinking at prerequisite "lower levels" (Fuys et al., 1988, p.4). They investigated the prerequisite reasoning abilities needed to successfully engage a logical-deductive system of thought. They described five levels that characterised the thinking of children as they become more sophisticated in their understanding of geometric relationships (Malloy, 2002). This is the most prominent feature of the theory. The levels describe "how one thinks about, rather than how much knowledge one has" (van de Walle, 2001, p.309).

According to van Hiele (1999, p.311), Level 1 (Recognition or Visualisation) starts with nonverbal thinking and the figures are judged by their appearance; in Level 2 (Analysis) figures are the bearers of their properties; in Level 3 (Informal Deduction) the properties of figures are logically ordered; in Level 4 (Deduction) students can manage implications with induction and they can write proofs by themselves and in Level 5 (Rigor) students can compare systems based on different axioms and can study various geometries in the absence of concrete models. The inability of learners to achieve *Recognition* (level 1) has resulted in researchers suggesting the addition of level 0 (Pre-recognition). Clements and Battista (1992, p.429) defined level 0 as "children initially perceive geometric shapes, but may attend to only a subset of a shape's visual characteristics and they are unable to identify many common shapes". Advancing from one level of thinking to the higher level depends on the student's experience and not the chronological order of the student's age (Abdullah & Zakaria, 2011). A child's adequate geometric experiences may either be in a classroom or elsewhere in order to move to a higher level of sophistication (Gujarati, 2014). It is the quality and nature of the experience in the teaching and learning program that influences a genuine advancement from a lower to a higher level.

The van Hiele theory was primarily directed at improving teaching as well as the geometric understanding by learners. This is achievable if instruction is organised in such a way that it took learners' thinking ability into account whilst new work is introduced. The model clarifies many of the shortcomings in traditional instruction and offers ways to improve it by focussing on getting students to the appropriate level to be successful in high school geometry (Pittalis et al., 2009). Pegg (1997) suggests that the levels have proved a useful tool in identifying the problems in students' understanding of certain geometrical concepts; secondly, in evaluating the structure or development of geometric content in secondary school textbooks and thirdly, in guiding the development of syllabi. The van Hiele theory is particularly relevant in South Africa, where mathematics remains a problematic learning area This is because as Fuys et al., (1988) suggest "its emphasis on developing successively higher thought levels appears to signal direction and potential for improving the teaching of mathematics" (p.191). According to Pegg and Davey (1998), the van Hiele theory is aimed at improving teaching by organising instruction to take into account learner's thinking. They continue to state that if the student's level of thinking is addressed in the teaching process, students will have the ownership of the encountered material and the development of insight will also be enhanced. For the van Hieles, the main purpose of the instruction is the development of such insight (van Hiele, 1986). The theory also offers a model of teaching that teachers can apply in order to promote their learners' levels of understanding of geometry (Atebe, 2008).

NCTM (1989), argues that "spatial understandings are necessary for interpreting, understanding and appreciating our inherently geometric world" (p.48). However, researchers (e.g., Burger, 1985; Fuys et al., 1988; Mayberry 1983; Renne, 2004; Usiskin, 1982) have indicated that learners are failing to learn basic geometric concepts including problem solving skills. In addition, "many teachers teach only a portion of the geometry curriculum that is available to them" (Clements, Battista & Sarama, 2001, p. 2). There is a belief that senior secondary school learners are underprepared for the geometry curriculum. For example, Usiskin (1982, p.29) commenting about the U.S. curriculum points out that "there is no geometry curriculum at the elementary school level. As a result, students enter high school not knowing enough geometry to succeed. There is a geometry curriculum at the secondary level, but only about half of the students encounter it, and only about a third of these students understand it". As stated earlier, the South African education system also has similar challenges. In senior secondary schools learners are expected to work with (1) a wide range of patterns and transformations and solve related problems and (2) describe, represent and analyse shape and space in two and three dimensions using geometry and trigonometry to achieve the learning outcome of space and shape. This curriculum for geometry consists of a mixture of unrelated concepts with no systematic progression to higher levels of thinking that are required for sophisticated concept development and substantive geometric problem solving.

This study was meant to address the deficiencies in senior secondary school geometry instruction in South Africa. This was done by focusing on an instructional framework based on van Hiele model.

THE INSTRUCTIONAL FRAMEWORK

An effective instruction is guided by general pedagogical approaches and specific instructional practices. For the study, an instructional framework was developed based on the van Hiele levels its implementation in the classroom. For the purposes of this study, this is referred to as van Hiele levels-based instruction. Malloy (2002) states that in implementing instruction based on the van Hiele framework, teachers have two tasks. Firstly, the teachers need to recognise and understand the van Hiele levels of their students, and secondly, they need to help students' progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. The NCTM (2000) states that "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p.12). This formed the basis for the development of the instructional framework. Furthermore, applying scaffolding techniques in instructional strategies is of great significance especially in the light of adoption of 'Inclusive Education' in South Africa's national education policies. Scaffolding in education involves optimal step-by-step pedagogical support to the learners which subsume, amongst others, the recognition of active role of prior knowledge, leading learners from the concrete to the abstract. That is, moving from simple concepts and principles to more difficult ones, through the use of effective visual support and visual organisers and engaging learners in appropriate and relevant learner activities.

van Hiele (1999) points out that school geometry has been following the axiomatically fashioned Euclidean geometry which requires the learners to think on a formal deductive level. This is not usually the case because learners lack the prerequisite understandings of geometry. This deficiency creates a gap between the learners' level of thinking and that required for the geometry that they are expected to learn. One of the characteristics of the van Hiele levels is that geometric experience is the greatest single factor that influences advancement through the levels. Activities that permit children to explore, talk about and interact with content at the next level, while increasing their experiences at their current level, have the best chance of advancing the level of thoughts. However, the van Hiele theory does not indicate what content to teach, but it does provide the thoughtful teacher with a framework in which to conduct geometric activities (van de Walle, 2004). van Hiele believes that development of learners' level of thinking is more dependent on instruction than on age or biological maturation and those types of instructional experience can foster, or hamper development (van Hiele, 1999). Instruction intended to foster development should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and concluding in summary activities that help learners assimilate what they have learned into what they already know (van Hiele, 1999). Rich and stimulating instruction in geometry can be provided through playful activities with mosaics and tangram puzzles (van Hiele, 1999). Children should be given ample opportunity for free play and for sharing their creations. Such play gives educators a chance to observe how children use the pieces and to assess informally how they think and talk about shapes. In solving puzzles, children work visually with angles that fit and sides that match. Children who use triangle grid to record solutions to puzzle become aware of equal angles in the grid and also of parallel lines. Activities using paper folding, drawing and pattern blocks can enrich children's store of visual structures. They also develop knowledge of shapes and their properties (van Hiele, 1999). As students see, touch, and manipulate shapes, they begin to develop spatial reasoning skills (Howse & Howse, 2014). According to van Hiele (1986), a suitable choice of exercises can create a situation for the learner favourable to the attainment of the higher level of thinking. A teacher beginning the teaching of geometry at a level that the learners are operating will inspire their confidence and the learners will try to understand the teacher (van Hiele, 1986).

The instructional framework developed by the present study provides activities starting at the visual level with an introductory game. According to van de Walle (2004), activities on grid papers are second best alternative to real physical objects and this helps the learners to do spatial explorations easily. The framework also has activities where the learners have to do the identification of different geometric shapes and figures from a collection of triangles and to recognise similarity between shapes in a grid paper. The other activities in the framework included a lot of sorting, identifying, and describing a variety of shapes. The framework also took note of the suggestion that spatial sense is enhanced by an understanding of shapes, what they look like, and even what they are named and the concepts of symmetry, congruence, and similarity will contribute to understanding our geometric world as suggested by van de Walle (2001). For the present research, the instructional materials were constructed using different sources such as textbooks, worksheets and past research materials and the adaptation of these were done in consultation with a researcher (Atebe) on van Hiele theory and the mathematics educators from the participating schools. Then piloting was done in one school in one session of geometry teaching and the materials were then modified to address the minor deficiencies.

METHODOLOGY

The sample consisted of two intact groups of Grade 10 classes from five schools. In all, there were 359 participants from five conveniently selected secondary schools from one Education District in South Africa's Eastern Cape Province. Of the total participants, 195 and 164 were assigned to the experimental and control groups, respectively. In this study, a quasi-experimental design was implemented to check the effectiveness of the instructional framework. The experimental and control groups were comprised of learners who were instructed with the van Hiele instructional framework and those instructed with traditional method, respectively. The experimental group was taught with the instructional framework by their respective mathematics educators. These five mathematics educators were trained on the van Hiele theory based instructional framework by one of the researchers. The instructional framework was implemented during the third term of the South African school calendar due to the fact that the work schedule of Grade 10 caters for geometry teaching in the third term. Pre-and post-tests were given to the participants before and after the five-week instruction to check the effectiveness of the instructional framework.

To achieve the main purpose of the study as whether the researchers' developed framework made any improvement in the levels of geometric thinking, it was imperative to measure the shift in performance of the experimental group before and after the intervention with the framework. The same test was administered as a pre-test and post-test on all the learners in the sample and a statistical analysis was conducted. The research instrument, the van Hiele Geometry Test, which was used to determine the van Hiele level of geometric thinking, which was used as the pre-test and the post-test, was done on topics such as basic geometric concepts like identification, classification and properties of triangles and quadrilaterals and angle measurement, angle sums of lines, triangles and quadrilaterals. The van Hiele Geometry Test that was used as a multiple choice test comprised 4 subtests. Each subtest consists of 5 items based on one van Hiele level. There were 20 items in the test, with item numbers 1-5, 6-10, 11-15, and 16-20 for testing learner's attainment of van Hiele levels 1, 2, 3, and 4 respectively. This test which was constructed by the staff of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project, developed by Usiskin (1982), and was adapted by Atebe (2008) with permission. The test was used among South African learners with Atebe's permission.

There was compliance with the use of informed consent form, guarantee of anonymity and permission from the relevant authorities as required and theses among others were produced to get the ethical clarence from the relevant authorities.

RESULTS

The grading of the van Hiele Geometry Test was done again using a second method which was based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22) to assign learners into different van Hiele levels.

Table 1. Learners' performance in the pre-test and post-test in terms of percentage mean score according to experimental group and control group in all schools

	Experimental group (N= 195)		Control group (N=164)		df	t-value	p-value
	% mean score	Standard deviation	% mean score	Standard deviation			
Pre-test	32.26	12.10	33.87	10.82	357	-1.322	0.187
Post-test	42.21	10.99	37.32	12.29	357	3.976	0.000

Table 2. Learners' performance according to experimental group and control group in all schools in the pre-test and post-test

	Number	Test	Mean	Standard Deviation	df	t-value	p-value
Experimental group	195	pre-test	32.26	12.10	194	-11.669	0.000
		post-test	42.21	10.99			
Control group	164	pre-test	33.87	10.82	163	-3.362	.001
		post-test	37.32	12.29			

Table 3. Percentage number of learners at each van Hiele level in the pre-test and post-test: All schools

Van Hiele levels	Percentage number of learners at each van Hiele level in all the schools in the pre- and post-tests			
	Experimental group		Control group	
	Pre-test	Post-test	Pre-test	Post-test
Level 0	56	26	56	47
Level 1	26	35	25	32
Level 2	17	38	18	20
Level 3	1	1	1	1
Level 4	0	0	0	0

DISCUSSION

It was evident from Table 1 and 2 that there was an increase in the percentage mean scores of both groups in the post-test. The experimental group's percentage mean scores increased from 32, 26% to 42, 21% and the control group's percentage mean scores increased from 33, 87% to 37, 32%. The percentage increase in the experimental group was higher than that of the control group. The paired samples t-test was used to find out whether there was a statistical difference between the two groups as the comparison was on the numerical information obtained from the same subjects under two surveys, namely, pre-test and the post-test. It was found that there was a statistically significant difference in the performance of the experimental group in the pre-test and the post-test.

As shown in Table 3, in the experimental group in the post-test, the percentage of number of learners at level 0 decreased from 56% to 26%, the percentage of number of learners at level 1 increased from 26% to 35% and the percentage of number of learners at level 2 increased from 17% to 38%. While in the control group in the post-test, the percentage of number of learners at level 0 decreased from 56% to 47%, the percentage of number of learners at level 1 increased from 25% to 32% and the percentage of number of learners at level 2 increased from 18% to 20%. The significant improvement in the performance of the experimental group having more learners at level 2 than at level 0 and level 1 in the post-test suggests that the van Hiele-theory based instruction for the experimental group had a more positive effect than those in the control group. It was also consistent with the study of Usiskin (1982) which stated that about a third of the students stayed at the same level or went down, about a third went up one level and about a third went up two or more levels. In general, it can be assumed that all schools benefitted out of the instructional framework through the considerable reduction of the percentage number of learners in the lower levels of the van Hiele levels (see post-test results in Table 3) and the statistical inference that there was a significant increase in the percentage mean scores in the experimental group.

The analysis of the levels of thinking in Table 3 showed that most of the learners were at level 0. This was an indication that the majority of learners had difficulty in identifying common figures and recognising figures in non-standard positions. Learners had an inadequate understanding regarding the identification of the geometric shapes using their properties and their orientation in space.

The low achievement at level 3 as seen in Table 3 shows that the learners are not ready for formal proof in Euclidean geometry as it used to be the level expected of senior secondary school learners. These findings lead to the importance on the delivery of instruction that is appropriate to learners' level of thinking. Many studies conducted in different parts of the world (Atebe, 2008; Clements & Battista, 1992; Fuys et al., 1988; Hoffer, 1981; King, 2003; Senk, 1985; Shaughnessy & Burger, 1985; Usiskin, 1982) highlighted that learners' poor performance in geometry are linked to the quality of classroom teaching. The concern in all the groups presented here is the number of learners at level 3 and level 4 as shown in Table 3. Only 1% of the learners in the entire sample of 359 learners in the present study were at level 3 and no one in Atebe's (2008) entire sample was on level 3 except for the South African subsample with 8% of them at level 4. Usiskin (1982) also noticed that 70% of his sample was operating at levels 1, 2 and 3. The majority of learners at level 0 indicated that learners had a weak knowledge in geometrical concepts. The main issue here is that most of the learners could not identify common figures and they could not recognise figures in non-standard positions.

The low 1% at level 3 and no one at level 4 indicated that learners in this study had difficulty in class inclusion of shapes, relationships between different shapes and properties of shapes. This was also consistent with the study of Mayberry (1983), where similar problems were noticed with the 19 pre-service elementary teachers in her study. Wirszup (1976) also had claimed that the majority of the high school learners were in the first level of development (level 1) while the course they took demanded level 4 thinking. It was evident that the majority of the learners in the study were also not reaching the level set by the curriculum, which expected the learners to be operating at level 3. None of the schools had learners at level 4 thinking on the van Hiele scale indicating that the learners were not ready for formal geometric proofs in Grade 10.

The majority of learners in the study at level 0 in the post-test, even though it had been considerably reduced from that in the pre-test, still indicates that learners are having conceptual difficulties. The increase in the percentage number of learners at levels 1 and 2 gives hope that a structured programme can raise the level of thinking.

In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum (Fuys et al., 1988; van de Walle, 2004). Haviger &

Vojkůvková (2014) suggest that secondary schools, with regard to their specialization, should determine what levels they want to achieve, and adapt the teaching of geometry to that goal. Malloy (2002) states that in implementing instruction based on the van Hiele framework, teachers need to recognise and understand the van Hiele levels of their students, and they need to help their students' progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. This was also supported by NCTM (2000). van Hiele (1999) points out that high school learners lack the prerequisite understandings about geometry and this lack creates a gap between their level of thinking and that required for the geometry that they are expected to learn.

Geometric experience is the greatest single factor that influences the advancement through the levels. Activities that permit children to explore, talk about and interact with content at the next level, while increasing their experiences at their current level, have the best chance of advancing the level of thoughts for those children (van de Walle, 2004). Instruction intended to foster development should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and concluding in summary activities that help learners assimilate what they have learned into what they already know. Rich and stimulating instruction in geometry can be provided through, among others, playful activities with mosaics and tangram puzzles (van Hiele, 1999). A study conducted by Siew et al. (2013) on students enrolled in Grade 3 showed that low ability students were observed to have the greatest improvement score compared to moderate and high ability students at the end of tangram activities. Promoting the transition from one level to the next should follow a five phase sequence of activities. Throughout these phases, the teacher has to plan tasks, direct children's attention to geometric qualities of shapes, introduce terminology and engage children in discussions using these terms and encourage explanations and problem solving approaches that make use of children's descriptive thinking about shapes. Cycling through these five phases with materials like the mosaic puzzle enables children to build a rich background in visual and descriptive thinking that involves various shapes and their properties (van Hiele 1999).

The results confirm the hierarchical nature of the van Hiele levels as more learners had answered the items correctly at level 1 and it was on a decline on moving to the higher levels. The learners in this study obtained the lowest percentage mean at level 3 which is consistent with the earlier studies of Usiskin (1982) and Atebe (2008), which concluded that the learners experienced more difficulty in attempting level 3 items than level 4 items. The percentage number of learners in each level proved to be the highest at level 0, followed by level 1 and level 2. There were no learners at level 3 and level 4 in all schools except in one school. These results are confirmatory to the earlier studies of Usiskin (1982) and Atebe (2008). Evidence that the sample from senior secondary school learners are not ready to do the formal proof that demands a thinking level of 4, is indeed a concern.

The significantly low percentages of learners at the higher levels of the van Hiele suggest that the learners experience difficulties in identifying and classifying shapes, properties of shapes and proof writing. This has also been noticed by past researchers such as Atebe (2008), Clements and Battista (1992), Fuys et al. (1988), Siyepu (2005) and Usiskin (1982). King (2003) found that there was significant difference in the performance of grade 6 learners after the intervention of the structured geometry course to the experimental group. Results from this study contradicts Genz (2006) and Halat (2007) who concluded that there was no difference detected in the acquisition of levels in schools using a curriculum based on the van Hiele theory (standards based curriculum) and schools using traditional curriculum (non-standards based curriculum).

Even though all the learners in the experimental group were taught with the instructional framework and there was a significant increase in their overall performance in the post-test, the performance in different schools and in individual learners were not the same. Many in the sample could not be raised to a level of thinking that is expected at the secondary school level. van Hiele also speaks of an unavoidable situation in class, where a group of learners having started homogeneously do not pass the next level of thinking at the same time. The difference in the percentage mean scores of the control groups between the pre-test and the post-test also shows that there was an improvement in the performance. This is also due to the instruction given to the learners in the traditional method. The improvement was not as significant as that of the experimental groups. Maturation and history of the learners also might have played a role in the increase in the scores of the learners in both experimental and control groups.

CONCLUSION

With reference to the research question investigated in this paper, it appears that the framework based on the van Hiele levels had a positive effect. The increase in the percentage mean score of the experimental group was higher than that of the control group. The paired samples t-test showed a statistically significant difference in the performance of the experimental group in the pre-test and the post-test. The significant improvement in the performance of the experimental group having more learners at level 2 than at level 0 and level 1 suggest that the van Hiele theory based instruction had a more positive effect than the traditional one. This indicates that a structured programme can raise the level of thinking. Through the considerable reduction of the percentage number of learners in the lower levels of the van Hiele levels, it can also be assumed that all schools benefitted out of the instructional framework in general.

The analysis of the levels of thinking showed that most of the learners were at level 0. This was an indication that the majority of learners had difficulty in identifying common figures and recognising figures in non-standard positions. This also indicates that learners had weak knowledge in geometrical concepts. The significantly low percentages of learners at the higher levels of the van Hiele suggest that the learners experience a lot of difficulties in identifying and classifying shapes, properties of shapes and proof writing. It was evident that the majority of the learners in the study were also not reaching the level set by the curriculum, which expected the learners to be operating at level 3. None of the schools had learners at level 4 thinking on the van Hiele scale indicating that the learners were not ready for formal geometric proofs in grade 10. The results also confirm the hierarchical nature of the van Hiele levels as more learners had answered the items correctly at level 1 and it was on a decline as it moves to the higher levels.

It was also noted that the performance in different schools and in individual learners were not the same even though all the learners in the experimental group were taught with the instructional framework and there was a significant increase in their overall performance in the post-test. Many of them could not be raised to a level of thinking that is expected at the secondary school level. This confirms van Hiele's observation that a group of learners in a class having started homogeneously may not pass the next level of thinking at the same time.

The control groups' difference in the percentage mean scores between the pre-test and the post-test shows that there was an improvement in the performance. This is also due to the instruction given to the learners in the traditional method although the improvement was not as significant as that in the experimental groups'.

Based on the above lessons learnt from employing the van Hiele based instruction, the following implications and recommendations are suggested for teaching and learning.

RECOMMENDATIONS

The geometric thinking level of the learners should be identified before the teaching program. It is recommended that for effective teaching in geometry to happen, teaching ought to start at the learners' level of thinking. For this, it is important that the levels are identified before commencing the teaching program. To improve geometry teaching, educators need to develop tasks or activities that help them better understand the nature of their learners' geometric reasoning and they also should have an understanding about research concerning such reasoning. Levels should be identified in earlier grades and appropriate experiences should be given in order to enhance achievement in geometry in senior secondary schools.

While drafting the curriculum, care should be given to arrange the contents in such a way that it should develop the geometric thinking from one level to the higher level. Changes in the instructional practices need to be coupled with the changes in the curriculum to observe the effects on learner achievement. Constructive activities should be encouraged. Learners should be made familiar with the techniques of drawing and folding for enhancing their geometric thinking. Higher levels of geometric thinking can be attained by the implementation of educator guided, learner centered, hands on instructional programme. It was evident from the higher percentage number of learners in the experimental group than in the traditional group in the post-test.

The process of gradually moving from the concrete to abstract and from passive to active learning under the guidance of the educators would make the learning of geometry more relevant and enjoyable to learners. Educators' main objectives should be to help learners to gain insight and understanding of the subject matter and consolidate their conceptual understanding.

It is necessary to design appropriate experiences for pre-service and in-service educators to familiarise themselves with the van Hiele theory so as to design and use appropriate material for instruction according to the levels. Van Hiele theory should be introduced into the curriculum of mathematics education. The initiatives aimed at revitalising teacher education and learner performance must also include efforts to improve classroom practices.

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