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This study was designed to investigate the relationship between the metacognitive awareness of university students and their solutions to the similar mathematical problem types. Participants were 97 freshmen from department of mathematics at a state university in Turkey. Two different scales were used for data collection: "Metacognitive Awareness Inventory" and "Mathematical Problem Types Test". The results showed that there was a significant positive correlation between the students' metacognitive awareness levels and their problem solving levels regarding routine and non-routine problems. There was no significant linear correlation between university students' metacognitive awareness levels and their problem solving levels for "separation", "combining", and "multiplication" in routine problems. Multiple regression analysis was used to test if the metacognitive awareness significantly predicted participants' levels of problem solving. The results of the regression indicated that metacognitive awareness significantly predicted problem solving levels and both predictors explained 45% of the total variance.

Keywords: problem types, metacognitive awareness, problem solving, problem solving level

INTRODUCTION

Problem solving ability in mathematics education is the point where knowledge, thinking ability and daily life come together. Due to this fact, problem solving has been emphasized in mathematics curricula in many countries since the 1980s. Problems are situations which arise in real life or in a scientific study and require selecting a strategy without straying from the core of the problem and they constitute an important part of an individual's mental activities (Biryukov, 2004). With the introduction of metacognition by Flavell (1979), the metacognitive parameters which influence the way individuals solve problems have been taken into account since 1980s and many scientists have combined metacognition and problem-solving and described metacognition as a key component of problem solving process (Schoenfeld, 1985). Problem solving must include both cognitive and metacognitive processess because a problem-solving individual has to select a strategy and think of alternative strategies as they come across difficulties and changing situations. However, cognitive processes such as the selection of suitable strategies are not only enough for the solution; a metacognitive monitoring that regulates these cognitive activities and monitors the efficiency of applications is also needed (Goos, Galbraith and Renshaw, 2000).

The problem types are generally classified in terms of the skill, the way of thinking and the effort they require for their solutions. According to this classification, problems are divided into two broad categories: routine and non-routine problems. Routine

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State of the literature

- Problem solving must include both cognitive and metacognitive processes.
- Mathematics problems are generally classified as routine and non-rountine problems.
- Metacognitive awareness is an individual's knowledge of his or her own metacognitive knowledge and metacognitive strategies.

Contribution of this paper to the literature

- It is an acknowledged fact that metacognitive awareness increases problem solving ability, but is this generalization valid for all problem types or is there any problem type which individuals with high metacognitive awareness are especially good at?
- In this study, the "Mathematical Problem Types Scale" which is intended to find individuals' success about routine and non-rountine math problems is developed by the researches.
- Is metacognitive awareness a meaningful precursor of solving mathematical problems types?

problems are mostly known as involving four operations in the literature as they are solved directly by using four operations. Their most important characteristic is that they require skills for four operations. In order to solve the non-routine problems, skills for four operations alone is not sufficient. The solution of these problems requires such skills as organization of data, classification, spotting relationships and it also requires doing certain activities consecutively. Unlike routine problems, these problems require using the necessary knowledge and skills for solutions in an unusual way (Öktem, 2009, p.27-28).
The term "metacognition" was introduced by Flavell in 1979. Flavell did research on children's long-term memory skills in 1976 and developed the term "metamemory". Metacognition, in its broadest sense, means an individual's being aware of their own thinking processes, planning these processes, arranging their thoughts about what they have planned and assessing the results (Ersozlu, 2008). Some research results revealed that metacognitive awareness could positively affect individuals' conceptual understanding (Sarıbas, Mugaloglu and Bayram, 2013). Costa (1984) describes metacognition as the ability to know what one knows and what one does not know, to be aware of the mental operations and strategies while solving problems and to assess and to meditate on our intellectual products. Metacognitive awareness is an individual's knowledge of their own metacognitive knowledge and metacognitive strategies. Flavell (1987) defines metacognitive awareness as thinking about thinking. According to Wilson (1998), metacognitive awareness means "*Metacognition is the knowledge and awareness one has of their own thinking processes and strategies and the ability to evaluate and regulate one's own thinking processes" (p.14).*

It is possible to find a lot of research in the literature which shows that metacognitive factors affect the problem solving process and performance. The findings from these studies show that there is a positive and meaningful relationship between metacognitive abilities and problem solving abilities and teaching these abilities increases problem solving performance (Schoenfeld 1982; Blakey and Spence 1990; Swanson 1990; Kapa 2001; Deseote,; Kramarski, Mevarech and Arami 2002; Teong 2003; Mohamed and Nai, 2005; Yimer and Ellerton 2006; Biryukov, 2004). It is an acknowledged fact that metacognitive awareness increases problem solving ability, but is this generalization valid for all problem types or is there any problem type which individuals with high metacognitive awareness are especially good at? The answer to this question is not known. The present study seeks an answer to this question.

Problem of the Study

What is the relation between metacognitive awareness of university students and their solving similar types of mathematical problems? In addition, answers to the following questions were sought;

(1) What is the relationship between university students' levels of metacognitve awareness and their solving mathematical problem types?

(2) What mathematical problem types did the students with different levels of metacognitive awareness show similarity in solving?

(3) Is metacognitive awareness a meaningful precursor of solving mathematical problem types?

METHOD

Participants

The sample of the research is the 97 first-year students from the department of mathematics in a state university. Of the 97 students, 57 were females and 40 were males.

Data Collection Tools

Metacognitive Awareness Inventory (BFE)

The Metacognitive Awareness Inventory, which was developed by Scraw and Dennison (1994) and translated into Turkish by Akın, Çetin and Abacı (2007), was used in order to determine the university students' levels of metacognitive awareness. This inventory uses a five-point Likert scale: from (1) for never to (5) for always. The inventory consists of two dimensions: the knowledge of cognition and the arrangement of cognition. These two dimensions are also divided into different factors. There are three factors under the knowledge of cognition dimension: declarative knowledge, procedural knowledge and situational knowledge and there are five factors under the arrangement of cognition dimension: planning, monitoring, assessment, elimination of erors and knowledge management (Akın, 2006: 159-160).

Mathematical Problem Types Test (MPT)

The MPT test was one developed by Yildirim (2010) to see what mathematical problem types the students can solve. The test consists of 5 multiple-choice items and 31 items with 8 different factors. It was determined that difficulty indices of the items in the scale varied between .22 and .84, discrimination indices between .20 and .84, item total correlations between .21 and .82 and item standard deviations between .37 and .50. Kr-20 value was calculated to be .93. For assesment of this test 3.0 points have been given for each true answer in the routine problems and 4.75 points for each true answer in the non-routine problems.

Data Analyis

The current study is quantitative in nature. Quantitative data were collected using two questionnaire. First, one is a Likert scale questionnaire of Metacognitive Awareness Inventory that ranges from one to five. Second one is a multiple choices test of Mathematical Problem Types that ranges from A to E. The quantitative data were collected with these two tools. The data were coded and entered into the Statistical Package for the Social Sciences V19.0 (SPSS) (IBM SPSS, 2010). Descriptive statistics, Pearson correlation and multiple regression analysis were used. Pearson Correlation and multiple regression analyses were conducted to examine the relationship between first year university students' metacognitve awareness and their problem solving levels regarding different type of problems.

RESULTS

The relationship between university students' levels of metacognitve awareness and solving mathematical problem types

The Pearson correlation coefficient was calculated using the total points from the Metacognitive Awareness Inventory and the Mathematical Problem Types test. There was a positive relation between levels of metacognitive awareness and solving mathematical problem types ($r= 0.673$, $p<.01$). Although the relation was average, it was still very close to high. Thus, it could be accepted that the relation between levels of metacognitive awareness and solving mathematical problem types was at a high level.

What mathematical problem types did the students with different levels of metacognitive awareness show similarity at solving?

Multiple variance analysis was done for each dependent variable to see if there was a difference between the groups. Whether there is a meaningful difference in terms of the students' levels of solving mathematical problem types according to their metacognitive awareness levels was examined. The differences for each problem type between the students with low, average and high levels of metacognitive awareness are as follows: No significant differences between the students with different levels of metacognitive awareness was found for "Separation" [$F(2\text{-}94)= .93$, $p> .01$] , "Combining" [$F(2\text{-}94)= 2.88$, $p< .01$], and "Multiplication" [$F(2\text{-}94)= 4.27$, $p>.01$] problem types. But there was significant difference for the following routine problem types: "Comparison"; [$F(2\text{-}94)= 6.10$, $p< .01$], "Dividing by grouping" [$F(2\text{-}94)= 24.39$, $p< .01$], "Division with remainder" [$F(2\text{-}94)= 11.18$, $p< .01$], and "Multistep" [$F(2\text{-}94)= 10.66$, $p< .01$]. When the routine problems were examined, there was a significant difference between the students with different levels of metacognitive awareness [$F(2\text{-}94)= 27.80$, $p< .01$]. As for the non-routine problems, there was a significant difference between the students with different levels of metacognitive awareness [$F(2\text{-}94)= 27.72$, $p< .01$].

Is metacognitive awareness necessary for the solution of mathematical problem types?

In order to determine whether metacognitive awareness is a meaningful precursor of solving mathematical problem types, simple regression analysis was done and prior to that a scatter diagram was examined to determine whether or not the relation between metacognitive awareness and levels of solving mathematical problem types was a linear one.

When we look at figure 1, it is seen that there is a direct relation between metacognitive awareness level and mathematical problem type solving.

According to the regression results regarding the prediction of the students' solving mathematical problem types from their metacognitive awareness levels, it can be seen that metacognitive awareness is a significant precursor of solving mathematical problem types.

Table 1. The Regression table regarding the prediction of the university students' solving of mathematical problem types from their metacognitive awareness levels

	Nonstandardized regression coefficients		*Standardized regression coefficients*				
	B	*sd*	*Beta*	*t*	*p*	*F*	*p*
(Constant)	-11. 891	8.032	.673	-1.481	.142	78.71	.000*
Metacognitive Awareness	.381	.043		8.872	.000*		

R=.673, R2=.453, Adj. R2=.447, **p*< .01

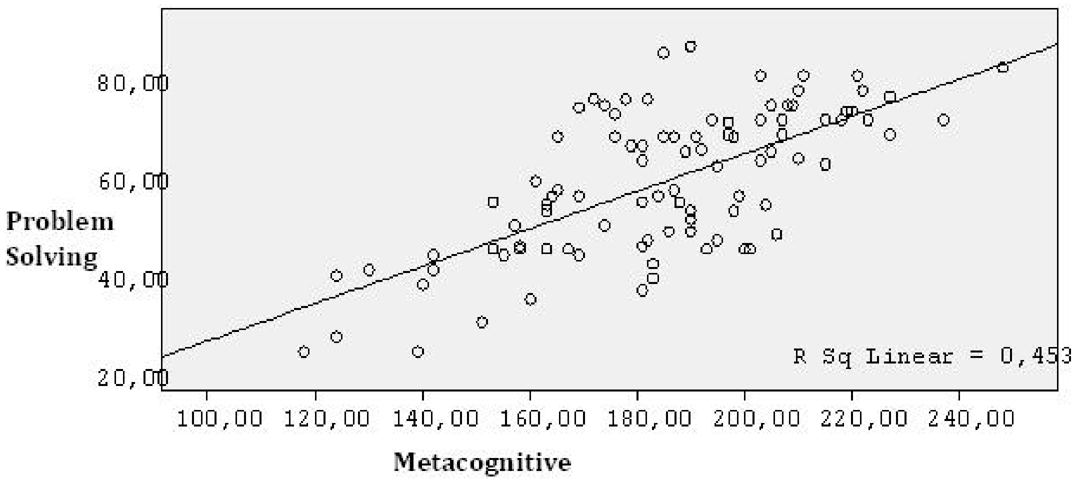

Figure 1. Scatter Diagram and Regression Line for metacognitive Awareness and Mathematical Problem Solving

DISCUSSION

According to the findings, there was an average and positive relationship between the university students' levels of metacognitive awareness and their levels of solving mathematical problem types. The fact that there is a near-high relationship between metacognitive awareness and solving mathematical problem types indicates that the level of solving both routine and non-routine mathematical problems increases as the level of metacognitive awareness increases. It also shows that the students' abilities to know their own thinking systems and their power to control them have an influence on the problem solving process and that metacognitive awareness increases the level of solving mathematical problems. This finding is consistent with the findings of earlier studies on the relation between metacognitive awareness and mathematical problem solving (Lucangeli and Cornoldi, 1997; Deseote et. al. 2001; Pugalee, 2001; Goos et. al. 2002; Teong, 2003; Mohamed and Nai, 2005; Balcı, 2007). When we examine the other finding of this study, we see that they do so in both routine, and non-routine problem types, though not in all types of routine problems. The students with different levels of metacognitive awareness did not show a significant difference in the routine problem types of separation, combining and multiplication. These three problem types are the simplest and require the least amount of resourcefulness. So, it can be claimed that metacognitive awareness is not effective with simple problems, but is effective with problems which require more skills, time, knowledge and thinking and which challenge students more. Finally, it can be claimed that the 45% of total variance regarding solving mathematical problem types can be explained by the students' metacognitive awareness and this is consistent with the findings of earlier studies on the relationship between metacognitive awareness and mathematical problem solving (Schoenfeld, 1985; Hartman, 1998). For future researches, since there is a significant relationship between metacognitive awareness and problem solving, the teaching of metacognitive skills should be given a priority in order to increase student success at problem solving. Using the process needs to be emphasized rather than being result-oriented in the problem solving process, so that what is missing from the behaviours of metacognitive awareness can be revealed or the particular behaviours which influence the problem solving process favourably can be determined and improved.

Authors' Note

This study was mainly based on the Master's thesis of the first author at Gaziosmanpasa University with Dr. Zehra Nur Ersözlü as the supervisor.

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