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Pre-service Physics Teachers' Comprehension of Quantum Mechanical Concepts

Nilüfer Didiş
Zonguldak Karaelmas Universitesi, Zonguldak, TURKEY

Ali Eryılmaz and Şakir Erkoç
Middle East Technical University, Ankara, TURKEY

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When quantum theory caused a paradigm shift in physics, it introduced difficulties in both learning and teaching of physics. Because of its abstract, counter-intuitive and mathematical structure, students have difficulty in learning this theory, and instructors have difficulty in teaching the concepts of the theory. This case study investigates students' comprehension of some fundamental concepts which are based on quantum mechanical postulates. The data of the study were collected by forty minute semi-structured interviews with two pre-service physics teachers conducted separately. In this study, qualitative analysis of pre-service physics teachers' dynamics of understanding showed that (1) students have insufficient conceptions that influence their descriptions and discriminations, (2) students' comprehension is indefinite, that means, they contain correct and wrong ideas simultaneously, influencing the students' use of different concepts interchangeably and making explanations and discriminations by intuitive reasoning, and (3) some of the conceptions of students are totally unscientific. In addition, students' comprehension lets only one way translation from mathematical to verbal.

Keywords: Physics Education, Quantum Mechanics, Comprehension

PEDAGOGICAL RESEARCH ON QUANTUM MECHANICS

Quantum theory is a physical theory which is constructed out of physical ideas, and expressed mathematically (Erkoç, 2006, p. XIII). The physical events of quantum theory are explained by mathematical tools. Merzbacher (1998, p. 1) defined quantum theory as a "theoretical framework within which it has been found possible to describe, correlate, and predict the behaviour of a vast range of physical systems, from particles through nuclei, atoms and radiation to molecules and condensed matter". It is regarded as a probabilistic physical theory (Busch, Lahti, & Mittelstaedt, 1996).

Correspondence to: Nilüfer Didiş, Ph.D student
The Middle East Techical University, 06531 Ankara-TURKEY
E-mail: *dnilufer@metu.edu.tr*

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Quantum theory changed all measurement techniques while passing from macro-world to micro-world in addition to changing interpretations in some parts of physics. The postulation approach draws upon the general ideas of scientific theories. For example, the first postulate of the quantum theory explains that any self-consistent and well-defined observable (such as energy, linear momentum, etc.) in classical physics corresponds with an operator in quantum mechanics (Liboff, 1998, p. 67). An operator should act on some functions, and the calculations of mathematical expressions which are measurement processes should be done to determine the behaviour of the particle. Going from classical physics to quantum physics is sometimes considered a paradigm shift in physics.

Quantum theory is taught as a compulsory course for physics majors in the department of physics at most

State of the literature

The research about quantum mechanics learning and teaching can be summarized into three domains:

- Difficulty in understanding quantum mechanical concepts,
- Elimination of difficulties in understanding quantum mechanical concepts,
- Examination of affect in learning quantum mechanical concepts.

Contribution of this paper to the literature

- This case study examines pre-service physics teachers' comprehension of the key concepts regarding the postulates of quantum mechanics and their problems with these concepts
- The examination of pre-service physics teachers' dynamics of understanding is important, since there is limited number of studies about pre-service physics teachers' conceptions of quantum mechanics.
- This study contributes to the literature by showing how pre-service physics teachers make sense of some quantum mechanical concepts. The results may be helpful in the improvement of content knowledge of pre-service physics teachers in quantum mechanics.

universities. It is generally considered to be a difficult course by many physics students because of its abstract nature and the requirement of complex mathematical formalism (Sadaghiani, 2005). In addition, many instructors also have some difficulties while teaching quantum theory, due to its new philosophy which is different from classical physics, its abstract concepts, and its lack of analogy and metaphors (Wattanakasiwich, 2005). Mathematics is the tool specially suited for dealing with abstract concepts of any kind, and there is no limit to its power in this field. For this reason any subject, not purely descriptive of experimental work, must be essentially mathematical (Dirac, 1958, p. viii).

The research about quantum mechanics learning and teaching can be summarized into three domains:

- *Difficulty in understanding quantum mechanical concepts:* While learning quantum mechanics, students usually do not have a chance to observe many of the phenomena and perform related experiments. Many studies revealed that students at both the university level and the upper high school level had difficulties in understanding and learning the concepts (Bao, 1999; Çataloğlu, 2002; Ireson, 2000; Ke, Monk, & Duschl, 2005; Morgan, 2006; Müller & Wiesner, 1999, 2002; Özcan, Didiş, & Taşar, 2009; Sadaghiani, 2005; Singh, 2001; Şen, 2000; Wattanakasiwich, 2005). Also some of the researchers found that students had misconceptions in quantum mechanics (Singh, 2001; Singh, Belloni, & Christian 2006; Styer, 1996).
- *Elimination of difficulties in understanding quantum mechanical concepts:* By considering the students' difficulties and misconceptions in understanding quantum mechanics, some researchers suggested some remedies to eliminate them by changing how quantum mechanics is taught through introducing new approaches and supporting the instruction with newly developed materials and media (Dobson, Lawrence, & Britton, 2000; Hadzidaki, Kalkanis, & Stavrou 2000; Michelini, Ragazzon, Santi, & Stefanel, 2000; Zollman, Rebello, & Hogg, 2002).
- *Examination of affect in learning quantum mechanical concepts:* In addition to these studies, students' motivation toward learning quantum theory (Didiş & Eryılmaz, 2007), and their attributions to success quantum mechanics (Didiş & Özcan, 2007; Didiş & Redish, 2010) were recently examined.

Misconceptions are stable, unscientific concepts of individuals. Misconceptions are unavoidable while learning quantum mechanics due to the difficulty of abstract concepts involved (Singh, Belloni, & Christian, 2006; Styer, 1996). Styer (1996) classified the misconceptions about quantum mechanical concepts into three major categories, which were (1) misconceptions regarding the idea of quantum states (wavefunctions, energy eigenstates etc.), (2) misconceptions regarding measurement, (angular momentum measurements, wave packets etc.), and (3) misconceptions regarding identical particles. Students' conceptual problems while learning quantum mechanics are not limited by misconceptions. One of the studies about students' conceptual difficulties was conducted by Müller and Wiesner (1999, 2002). The researchers investigated pre-service physics teachers' conceptualizations of atoms, permanent localization and the Heisenberg uncertainty principle. They focused on students whose major was not physics and thus they never had the chance for learning quantum physics conceptually. In Ireson's (2000) study, a 40- item questionnaire was given to 342 students to examine the students' understanding of quantum phenomena. The study showed that students could not interpret quantum theory. Sadaghiani (2005) investigated students who did not have a functional understanding about probability and related concepts and who had problems with some terminologies, sometimes confusing them with each other. Wattanakasiwich (2005) explained that the reason for students' difficulties in conceptual understanding was their lack of physics knowledge, so students did not understand the mathematical solutions conceptually, but only memorized the solutions in probability concepts.

In summary, students' conceptual problems in understanding are examined by considering visual and mathematical problems in understanding quantum mechanics since conceptual problems are not totally independent of them. In order to fix the conceptual problems in an effective way, these problems should be examined carefully. Problematic conceptions and their relations with other concepts can be investigated with a different research design. Qualitative investigation of students' comprehension of these concepts is one of the important ways to describe and explain the problems which prevent their understanding of the physical, mathematical and philosophical nature of the theory. Since it gives a chance to a researcher to explain "how" and "why" something occurs. This case study examines pre-service physics teachers' comprehension of the key concepts regarding the postulates of quantum mechanics and their problems with these concepts. In this regard, the research questions of the study are as follows: (1) How could pre-service physics teachers explain operator, observable, eigenvalue and interrelated concepts? (2) How could they relate and differentiate these concepts? (3) Could they translate the verbal expressions of these concepts to mathematical ones, or vice versa?

THEORETICAL FRAMEWORK

Comprehension of Concepts

A concept can be defined as "a set of specific objects, symbols, or events which are grouped together on the basis of shared characteristics and which can be referenced by a particular name or symbol" (Merrill & Tennyson, 1977 as cited in Smith & Ragan 1999, p. 179). Concepts are like "vehicles of thought" (Harre, 1966, p. 3). They are "fundamental agents of thought for human beings" and are used for indication of both mental constructs and identification of public entities (Klausmeier, Ghatala, & Frayer, 1974, pp. 1- 4).

Concepts may be abstract or concrete. Learning concrete concepts is achieved by experiencing their physical characteristics in daily life. Learning them is easier than learning abstract concepts. Learning the abstract concepts takes place in understanding the definitions of the concepts. In quantum mechanics, concepts are generally abstract. Students have no sensory experiences about them, so understanding of the definitions of the concepts is important for learning. Since memorization of the definition of a concept is easier than the understanding the physical meaning of an abstract concept, students sometimes prefer this easy way. Thus, the definition of a concept may be learned by memorization. However, "learning of a concept" and "learning of a definition of a concept" are not the same thing as noted by Smith and Ragan (1999, p. 179). Furthermore, unlike when "learning of a definition of a concept", concepts are related with other concepts when "learning of a concept". This was also explained by Halloun (1998) in such a way that isolated concepts were "meaningless and useless" unless they were related to other concepts.

Comprehension is a cognitive ability which is defined as the ability to grasp the meaning of material. It may be showed by translation of material to different forms (i.e. mathematical to verbal), by interpretation (i.e. explaining) and by estimation (i.e. predicting the effects) (Gronlund, 1971, p. 48). Intellectual abilities and skills which were emphasized in schools and colleges mainly focused on comprehension (Bloom, 1956, p. 89). Some behavioural terms were presented for specifying the meaning of comprehension of students by Gronlund (1971, p. 529), and Gronlund and Linn (1990, p. 507). "To convert, to defend, to distinguish, to estimate, to explain, to extend, to translate, to generalize, to give example, to infer, to paraphrase, to predict, to rewrite and to summarize" are some behaviours which indicate students' comprehension.

Quantum Mechanical Concepts

In classical mechanics, a measurable quantity is explained with a value in phase space. However, in quantum mechanics, it is explained in "Hilbert space", which is an n-dimensional vector space and its elements are vectors represented by the elements of a matrice. The postulate of quantum mechanics which is about measurement "any self consistently and well-defined observable in physics (such as, energy, linear momentum, etc.), there corresponds to a Hermitian operator in quantum mechanics" (Liboff, 1998, p. 66) indicates the transition from phase space to Hilbert space. The first key concept of the postulate is "observable", which means any physical quantity that can be measured, such as position, linear and angular momentum, and energy. A physical quantity is sometimes called a "dynamical quantity" (d'Espagnat, 2003, p. 46) which is liable to change in time. "Operator" is another key concept which has an important role in mathematical formulations (Hameka, 2004, p. 66). They are "set of instructions which were defined for some vector space, for changing one vector belonging to the space" (Anderson, 2005, p 86). Operator formalism provides the tools to construct in a simple and efficient way the solutions for the time dependent Schrödinger equation (Merzbacher, 1998, p. 41). Operators are linear functions in Hilbert space and they are represented by square matrices. An important class of operators is "Hermitian operator", which is also a linear operator, and its own ad-joint (that is $b_{ij}=b^*_{ji}$, in matrix representation). In quantum mechanics, every physical observable may be represented by a Hermitian operator (Hameka, 2004, p. 66).

Energy is a concept for all physical systems. The "Hamiltonian operator" is an important and widely used Hermitian operator (since its all eigenvalues and expectation values are real) in quantum mechanics that corresponds to the total energy of the system. An "eigenvalue" corresponds to a measured value of an observable and an "expectation value" corresponds to a statistical average value of measurement values. The Hamiltonian operator is a good subject for studying comprehension of the operator, observable, and eigenvalue concepts in the Schrödinger equation. Operator formalism is used because there is no way to predict measured values in quantum mechanics. The total energy of a system can be defined in terms of its kinetic and potential (Coulombic, relativistic interactions etc.) energies. For every physical particle in quantum mechanics, a Hamiltonian operator can be defined corresponding to the total energy of the particle.

METHODOLOGY

Data Collection

This is a case study which is known as an "in-depth study" that examines real life contexts that reflect the perspective of the participants (Gall, Gall, & Borg, 2007). The results of a case study does not provide generalizability to population, however, it may provide analytical generalizability for theories (Yin, 1994). It may generate specific hypotheses to be tested later by controlled-intervention research (Lising & Elby, 2005) and suggest a model (Yin, 1994). By this way, a rich picture of how students construct meaning on concepts can be identified. For this reason, this research focused on the comprehension of two pre-service physics teachers who were purposively selected. During data collection, data saturation was considered. The data reached saturation in almost forty minutes in both of the interviews.

Description of the Course Setting

Quantum Physics (PHYS 300) is a compulsory course for physics and physics education majors at the Department of Physics at the Middle East Technical University (METU). It is a prerequisite course for the Quantum Mechanics (PHYS 431) course and starts with the fundamental experiments of quantum theory, provides a conceptual framework for the theory by examining the postulates of quantum theory, probability, wavefunctions, the time dependent and time independent Schrödinger equations, harmonic oscillators and tunneling concepts etc. The main instructional methodology for teaching the concepts is lecturing enriched by solving mathematical problems. In addition to the lecture hours, extra problem solving hours are provided for the students. In lectures, concepts are presented, and some examples in the textbook are explained by the instructor. In the problem solving hours, various types of textbook problems are solved by the course assistant(s). Also, almost every week, homework problems are given to students to practice. It is a three-credit course and the textbook for the course is Liboff's (1998) "Introductory Quantum Mechanics". Almost sixty students take Quantum Physics each semester.

Participant Selection

The participants of this study were two pre-service physics teachers (physics education students) who have just completed the Quantum Physics course, at METU. These students were selected by considering their academic grades of in the Quantum Physics and Modern Physics courses. The grades range from AA to FF, and grade CC is accepted as average grade at METU. Both of the participants were average level students, by this way threats due to being un/successful were controlled. The participants of the study had attended the Quantum Physics course in the spring term. In the next fall term, they attended the Quantum Mechanics course. One of the students is male and the other is female, and they are also the same age.

Interview Procedure

After the students were selected, pre-interviews (pilot interviews) were done to check the communication between participants and interviewer. By doing so, missing data due to communication skills has been controlled.

Interview questions were constructed by considering the literature and personal experiences (see Appendix). The questions in the interviews for this study were firstly tested by one of the researchers who had taught a graduate level quantum mechanics course and wrote a quantum mechanics textbook (Erkoç, 2006). Then, they were examined by other two physics and physics education professors both in terms of content and appropriateness. The semi-structured interviews were conducted with each student in forty minutes. The interviews started with requesting students to explain (describe, give examples etc.) basic concepts, then continued with asking them to differentiate and relate the concepts. Next, translations of basic concepts were identified by the "think aloud" procedure.

The data collection procedure was the same for both of the participants. Both interviews were conducted by one of the researchers in a classroom which is a familiar environment for students. Both interviews were conducted on two consecutive days, and they were in the pre-service physics teachers' native language, Turkish. The interviews were recorded by video-camera with the consent of the interviewees, and notes were also taken during interviews. Some probe questions were asked to get deeper insight about explanations of them. As much as possible, at the end of the each question students were also asked to paraphrase their answers to prevent confusion. At the end of the interviews, the questions were reviewed by the interviewer, and the interviewee approved his/her answers.

Reliability and Validity

During the study, reliability and validity issues for qualitative studies were considered as explained by Yıldırım and Şimşek (2005) and LeCompte and Goetz (1982). Internal reliability precautions were provided by using mechanical devices, including multiple researchers into the study, peer examination of the data, presentation of the obtained data in a descriptive approach (p. 261-262). And, internal validity was provided by member checking and triangulation of results (Erlandson, Harrison, Skipper, & Allen, 1993).

Data Analysis

After the data collection, the researchers analyzed the data separately, so as to not affect each other at the first stage, and then compared the findings. Analysis of the data were performed to determine comprehension, which includes the explanation, discrimination, relation and translation of observable, operator and eigenvalue concepts. Subjective interpretation threats for this qualitative study were controlled by the examination of data and the findings by the authors of the research.

RESULTS

Explanation: Description and Giving Examples about Concepts

In the interview process, students (St1= 1. student, and St2= 2. student) were asked to explain what operators and observables were and to give examples of them. St1 explained *"an operator is a set of mathematical operations and energy and momentum are examples for operators"*. St2 had no definite conception about what an operator is. St2 only said *"… operators had some properties such as commutation, association, and they were linear… and energy and momentum are operators"*. Both of the students have partial knowledge about operator concept since the students' explanations of this concept show their insufficient understanding. St1 did not consider the operators in vector space, St2 did not indicate what they really were.

Observable is another key concept in this context. St1 defined observable as *"measurement result"*, confusing it with the eigenvalue concept. The student was asked to clarify this statement. The student's explanation was as follows:

St1: (thinking) … *We applied an operator and we obtained something. For example, we applied the Hamiltonian operator to a wavefunction and we obtained some values for energy, these are observable… We cannot determine observables without computation.*

St2 described observable as *"measurable"*. This may be accepted as a good explanation for observable. Both of the students gave the same examples "energy and momentum" for the observable concept. Due to these answers, in both interviews the students were asked whether "energy and momentum" were operators or observables. Although St1 gave a good explanation about observables, St2 answered as *"Mmm… intuitively… operators are theoretical, they provide calculations, they provide a conclusion, and observables are the conclusions of them"*. Students think that an observable is obtained by using an operator, however, it is not. An observable in a classical mechanical system is known and its operator is defined in a quantum mechanical system. After application of an operator to a wavefunction (measurement), possible values, eigenvalues, are obtained. However, by investigating with some probe questions, it is seen that students tend to use observable and eigenvalue concepts interchangeably as giving the same examples correctly by in intuitive reasoning.

Discrimination and Relation of the Concepts

In the first part, students' explanations showed that they have difficulty distinguishing observable and eigenvalue concepts. After we obtained some background knowledge of students' conceptions about operator and observable concepts, the next questions in the interviews were about "Hamiltonian operators" and "Hermitian operators". Students were asked to explain what they were. The Hamiltonian is a good case for investigating students' comprehension of observables, operators, and eigenvalues. Although these concepts were explained to students in the lectures, they are generally unfamiliar with their conceptual meanings. These concepts must be discriminated in order to understand the overall idea of quantum mechanics.

First, students were asked to explain the Hamiltonian operator. One of the students explained that *"it is used for energy to obtain eigenvalue"*. Although this student explained observable as an eigenvalue in the previous questions, the same student used the eigenvalue concept in right way to explain Hamiltonian operator. This result indicates the clues of not having mostly correct and structured models of these concepts, not making sense of the concepts, and memorizing some basic ideas about concepts.

Then, the students were asked what a Hermitian operator was. Both of the students could not remember what a Hermitian operator was. Also, it was requested to relate Hermitian and Hamiltonian operators. Interestingly, both students said a Hermitian operator is *"an operator for specific observable"*. However, Hermitian operator is a class of operators it is not specific for observables like the Hamiltonian operator. In other words, operators that we used in quantum mechanics are already Hermitian operators which are also linear. Students' other explanations are the following:

St1: … *I do not remember the Hermitian operator exactly… I know they are not same as the Hamiltonian operator… Both of them are operators. Both of them act on a system differently, I remember that. We find energy eigenvalues with the Hamiltonian, but I do not remember what we find with Hermitian operators… They are different from each other but I do not know why.*

St2: (smiling) … *Maybe I am confused, both of them were represented by "H"… I suppose, the explanation of the Hamiltonian operator is a bit longer than a Hermitian operator… They are not same because I remember that we learned them in Quantum Physics lecture separately… We learned them under different titles.*

Although, St1 explained that they were different things, the student could not show the difference between them, and St2 said also similar things.

Translation

In this study, students' translation of both verbal information into mathematical form, and mathematical information into verbal statements were investigated for the "operator" concept in quantum theory. More specifically, students' verbal and mathematical explanations about the Hamiltonian operator were examined.

It was seen that students could give example "energy" and "momentum" as examples of the operator concept. When students were asked to state energy and momentum operators mathematically, both of them stated as *"I could not state the Hamiltonian operator in the mathematical form"*. Then, time dependent and time independent Hamiltonian operators in mathematical form were presented to students but their names were not given. Then, students were asked to identify the operators and interpret the mathematical statements verbally. Both of the students called them *"Hamiltonian operators"* and added that *"this represents the total energy of the system"*. One of the students reached "total energy" conception step by step, in a suspicious way. The student first stated *"Hamiltonian operator represents kinetic energy, there is p here, it is momentum used in kinetic energy expression (p^2 / 2m)… Mmm… But here is V. This is potential energy… Both of the energies…"*

The explanations showed that, in the operator concept, students had difficulty in translating verbal statements into mathematical form. However they could translate into verbal statements, recall the name of the operator correctly by interpreting the descriptors in the equation(s), and they can interpret the mathematical structure.

Table 1 summarizes the whole structure of the comprehensions of two pre-service physics teachers considering their explanation, discrimination, relation and translation of "operator, observable and eigenvalue" concepts.

In addition, some other conceptions were identified in the interviews, for example, students have no clear idea about what particles are (are they atoms, photons, electrons, protons etc?). In other words, the "particle" concept is ambiguous to them. The other conception is also an unscientific conception, and it overgeneralizes

Table 1. Students' Comprehension of Operator, Observable and Eigenvalue Concepts by means of Behavioural Elements

Behavioural Elements	Student Conceptions
Explanation	*Operator is a set of mathematical operation*
(description, giving example)	*Commutation, association and linearity are characteristics of operators*
	Energy and momentum are examples for operators
	Observable is a measurement result
	We apply an operator to a wavefunction and we obtain something, these are observables
	We cannot determine an observable without computation
	Observable is a measurable
	Energy and momentum are examples for observables
	Hamiltonian operator is used for energy eigenvalue
	Hermitian operator is an operator for a specific observable
Discrimination and Relation	*Hamiltonian operator is different from Hermitian operator*
	Explanation of Hamiltonian operator is longer than Hermitian operator
	Hamiltonian operator and Hermitian operator are represented by H
Translation	*I cannot state the Hamiltonian operator in the mathematical form*
	The hamiltonian operator represents the total energy of the system (after seeing the mathematical form)
	The hamiltonian operator represents both of the energies (kinetic and potential)

the measurement process in quantum mechanics by stating *"We could apply Hamiltonian operator to any other things (not wavefunction, Ψ), this depends on what we will measure"*.

CONCLUSION, LIMITATIONS, AND IMPLICATIONS

Comprehending operators, observables and eigenvalues have been examined in the boundary of behavioural elements which are explanation (description, giving example), discrimination, relation, and translation. As it is presented in the table, pre-service physics teachers have some insufficient and unscientific conceptions, and indefinite comprehensions. The results are similar with the Wattanakasiwich's (2005) study which indicated students' understanding lacks physics knowledge and they had problems with some terminologies in probability concept. Also, Sadaghiani's (2005) study indicated students confused the terminologies as identified probability, operators, wave function and uncertainty concepts. Halloun (1998) explained the importance of students discriminating the concepts like physicists would, and not using them interchangeably. However, the results showed that the pre-service physics teachers did not identify some concepts in description and understanding levels. These results are also as similar with the findings of Singh (2001), which identified students' lack of discrimination of quantum mechanical concepts in mathematical problem solving such as not understanding the role of the Hamiltonian operator in the time development of the system and the lack of stating the significance of the operators which commute this operator. Having insufficient conceptions and indefinite comprehension correspond with the situations mentioned by Singh (2001) in three categories, which are based on "lack of knowledge related to a particular concept, knowledge that is retrieved from memory but cannot be interpreted correctly, and knowledge that is retrieved and interpreted at the basic level but cannot be used to draw inferences in specific situations". In this study, there are some clues about having insufficient conceptions triggering indefinite comprehension. In addition, problems in translation are also not independent of conceptual sufficiency. That means, because of insufficiency in the conceptual part, students explain the concept by trying to find clues in the mathematical form of it.

As a conclusion, in this study, pre-service physics teachers' dynamics of understanding can be summarized as (1) students have insufficient conceptions that influence their descriptions and discriminations, (2) students' comprehension is indefinite, that means, they contain correct and wrong ideas simultaneously, influencing students' use of different concepts interchangeably and making explanations and discriminations by intuitive reasoning, and (3) some of the conceptions of students are totally unscientific. In addition, students' comprehension lets only one way translation from mathematical to verbal.

This study which examined dynamics of understanding of two pre-service physics teachers concludes that they have difficulty in learning quantum mechanical concepts even if they pass the course. One of the reasons of conceptual problems of the students may date back to the high school physics. Fortunately, in new Turkish Physics Curricula, it was aimed to explain some concepts of quantum mechanics conceptually to science major students in Modern Physics Units in 11 and 12 grades (Ortaöğretim 11. Sınıf Fizik Dersi Öğretim Programı, 2008; Ortaöğretim 12. Sınıf Fizik Dersi Öğretim Programı, 2009). This change in high school physics curricula may help students start to make sense of these concepts in high school as it was mentioned in Şen's (2000) study.

Another issue which has influence on students' understanding is quantum mechanics examinations mainly require higher order mathematical knowledge instead of comprehension of "concepts". This causes students to "ignore" the fundamental concepts that form the basis of quantum theory. Although the results regarding students understanding cannot be generalized, they may reflect understanding of many other students in similar lecture settings. By considering the students' comprehension, concepts should be constructed well before solving quantum mechanics problems, since the lack of physics knowledge may lead students to memorize solutions (Wattanakasiwich, 2005). Conceptual problems in students' comprehension are not totally apart from the mathematical nature of quantum mechanics. At this point, students should try to see whether the difficulty is in physics or in mathematics (Erkoç, 2006, p. XIII). For this reason, the instructors should be more interested in the explanation of concepts by redesigning the instructions; considering students' dynamics of understanding, and examinations should be revised measuring conceptual understanding better.

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Appendix: Interview Questions

1. What do you know about Quantum mechanics postulates?
2. a) What does "operator" concept mean in physics?

   b) What are the tasks of operators in Quantum mechanics?

   c) How do the operators act on a system?

   d) Which operators do you know? Give example.
3. a) What does "observable" concept mean in physics?

   b) Which observables do you know? Give example.
4. What are the differences between "observable" and "operator" concepts?
5. What is a "Hamiltonian operator"?
6. What does a "Hermitian operator" mean?
7. a) How can you relate Hermitian and Hamiltonian operators?

   b) Are there any relationships between them?
8. a) Can you state some operators mathematically? *(if the students cannot state "Hamiltonian operator", ask 8.b)*

   b) OK, Can you state Hamiltonian operator mathematically? *(if the students cannot state, ask 8.c)*

   c) OK, what are the operators below? *(if the students can call and explain as "Hamiltonian operator", ask 8.d)*

   $\{H\} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$, $\{H\} = i\hbar\frac{\partial}{\partial t}$

   d) What is the task of hamiltanion operator?

   e) What are the descriptors in the equation that define the system? (by showing time independent Schrödinger equation)

   f) What is the observable of Hamiltonian operator?